

**PRICING AND COMPETITION BETWEEN
HETEROGENEOUS AUCTION SITES**

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FTC Roundtable on the Economics of Internet Auctions

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PRESENTATION OUTLINE

1. Prior Literature.

2. Modeling ingredients.

3. Game time-line and some analytics.

4. Three Examples.

4.1. Auction Hosting Site Monopoly, uniformly distributed seller costs.

4.2. Auction Hosting Site Monopoly, exponentially distributed seller costs.

4.3. Auction Hosting Site Duopoly, exponentially distributed seller costs.

5. Work in Progress/Future Work.

PRIOR LITERATURE

— **Seller to Buyer interaction - “traditional” auctions and bidding literature.**

McAfee and McMillan (1987), Klemperer (1999), Wolfstetter (1996), Krishna (2002).

Ashenfelter and Graddy (2003), Bajari and Hortacsu (2004).

— **Competing sellers literature.**

McAfee (1993), Peters (1997), Peters and Severinov (1997).

Hernando-Veciana (2005), Peters (2001).

Burguet and Sakovics (1999), Schmitz (2003).

Anderson, Ellison, Fudenberg (2004), Ellison, Fudenberg, and Mobius (2004).

Damianov (2005), Parlane (2005).

PRIOR LITERATURE . . .

- **Platforms/Two-sided markets.**

Rochet and Tirole (2004).

- **Differentiation of Internet retailers.**

Lowest price sellers are not the highest volume ones (Brynjolfsson and Smith, 2000).

Marketing impact as in brick-and-mortar (Clay, Krishnan, and Wolff, 2001).

Customer-level evidence of differentiation (Smith and Brynjolfsson, 2001)

Down-sloping demand curves and cross-elasticities (Chevalier and Goolsbee, 2003).

See Ellison and Ellison (2004b) and Ellison Ellison (2004a) for more.

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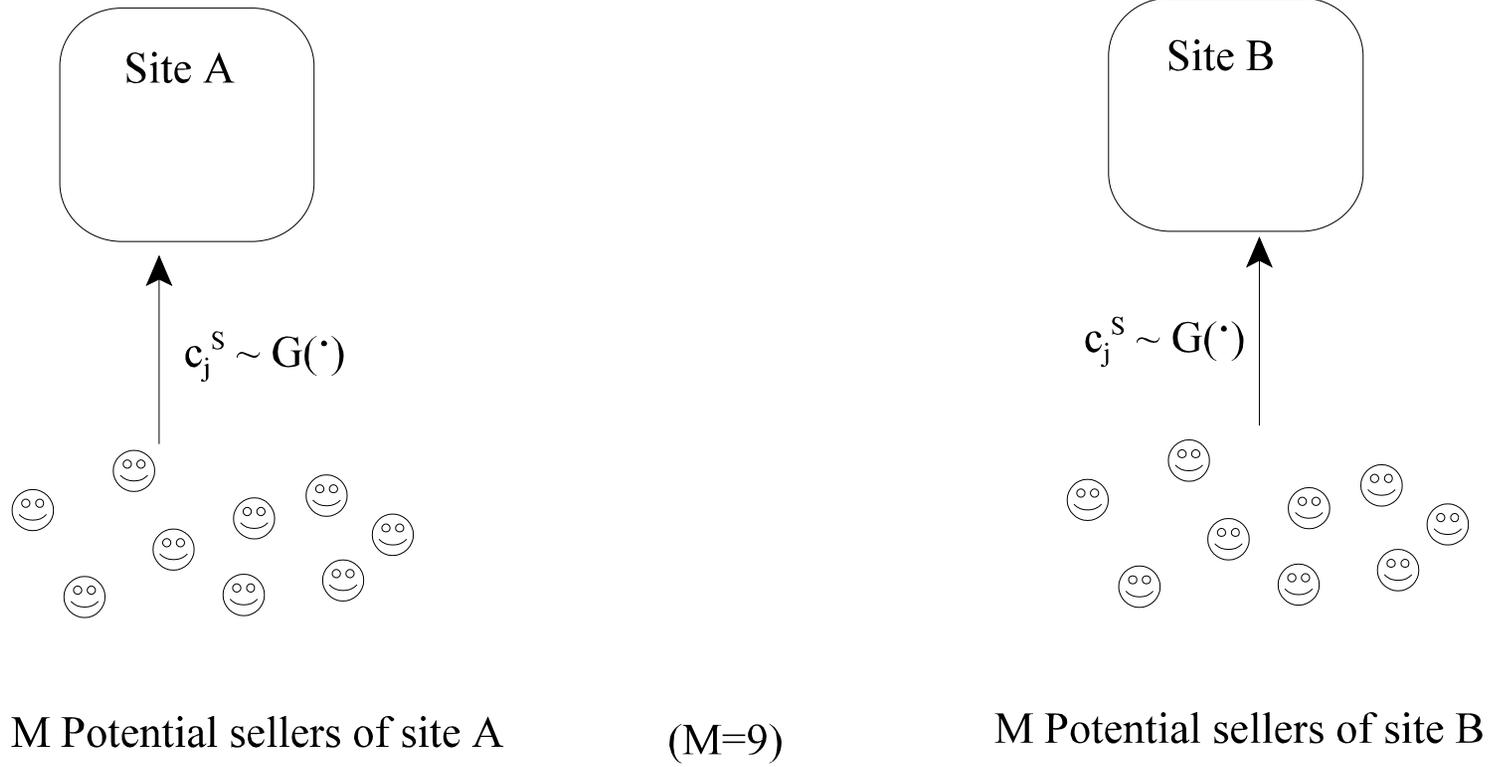
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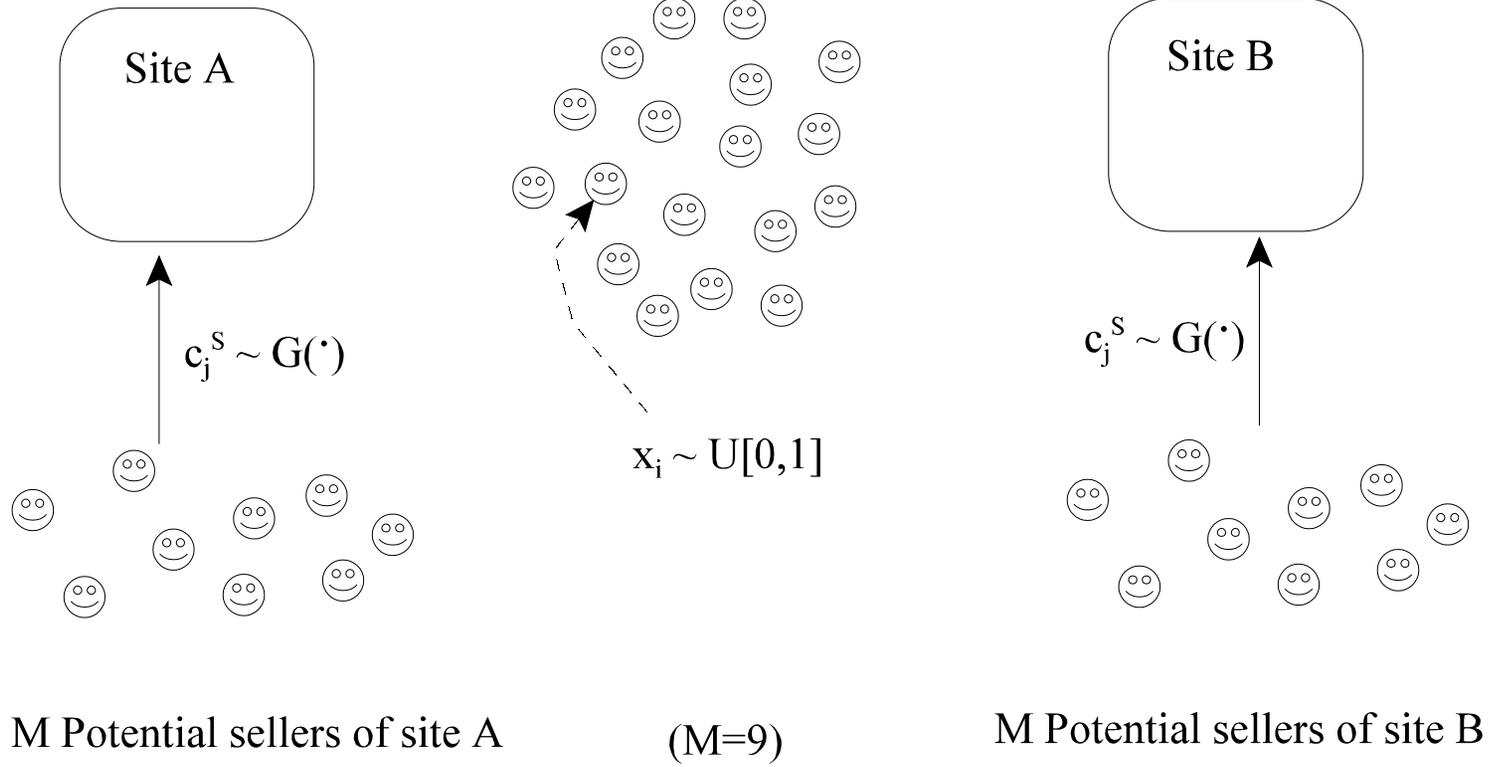
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SITES AND SELLERS

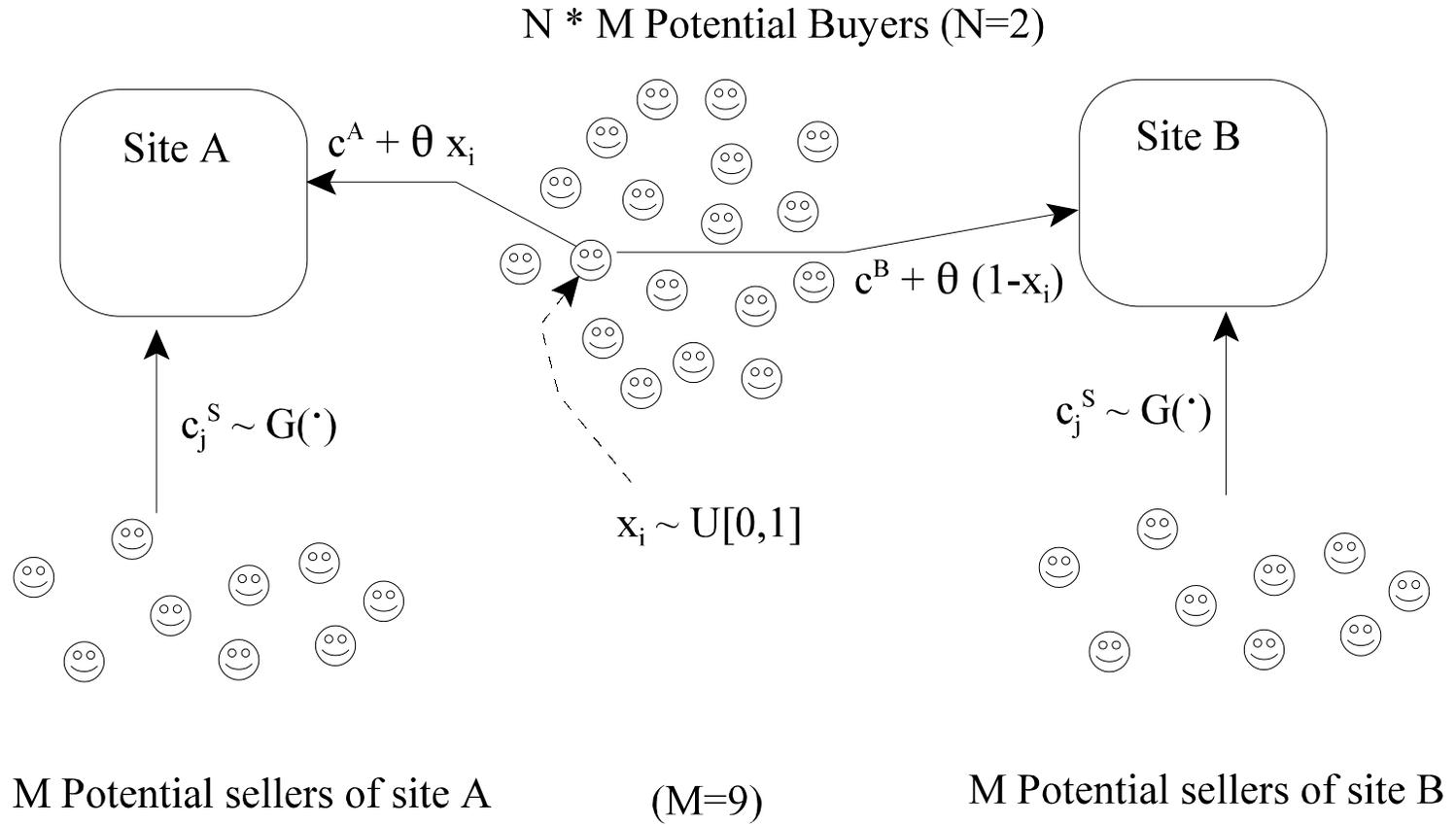


BUYERS

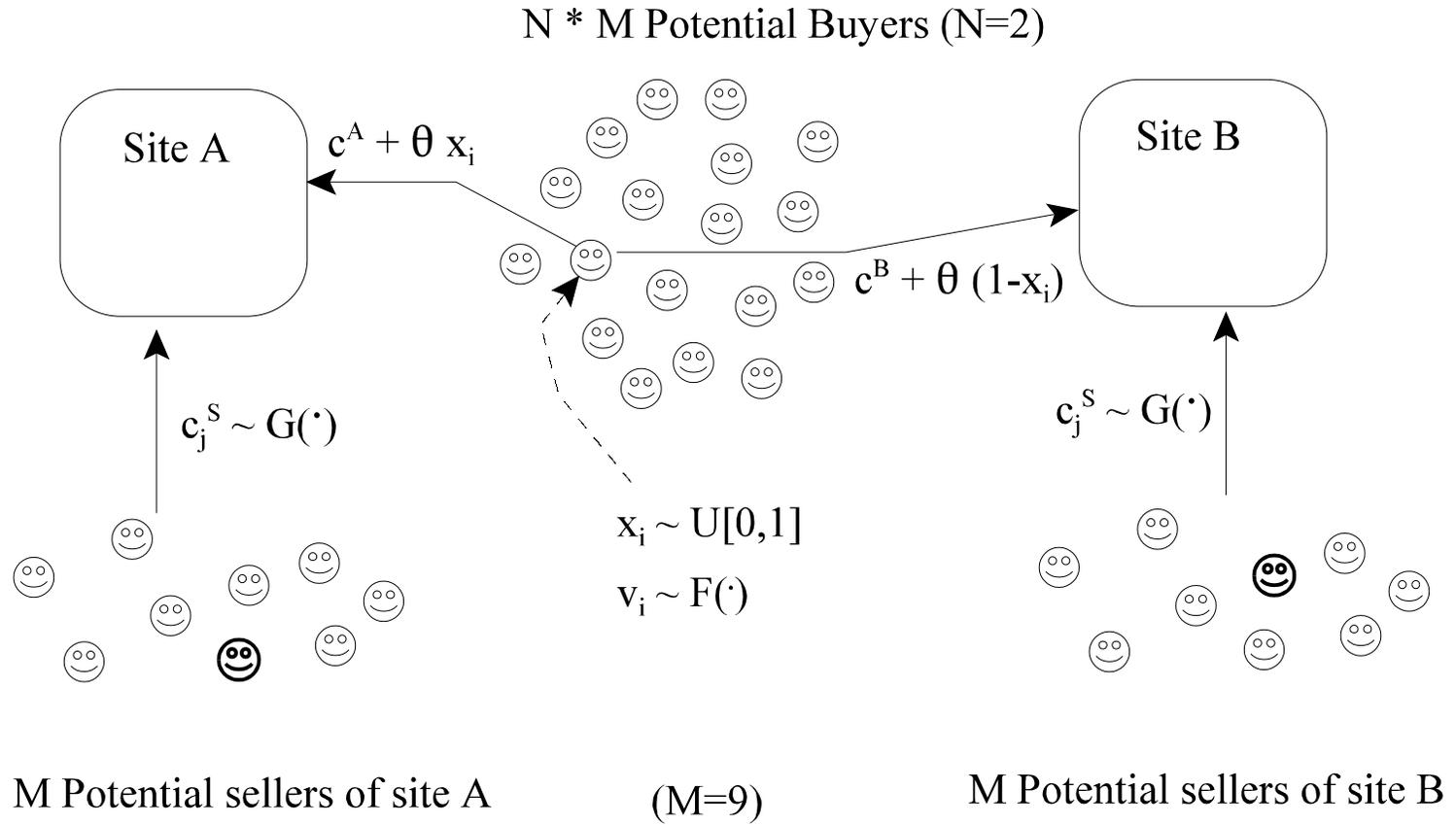
$N * M$ Potential Buyers ($N=2$)



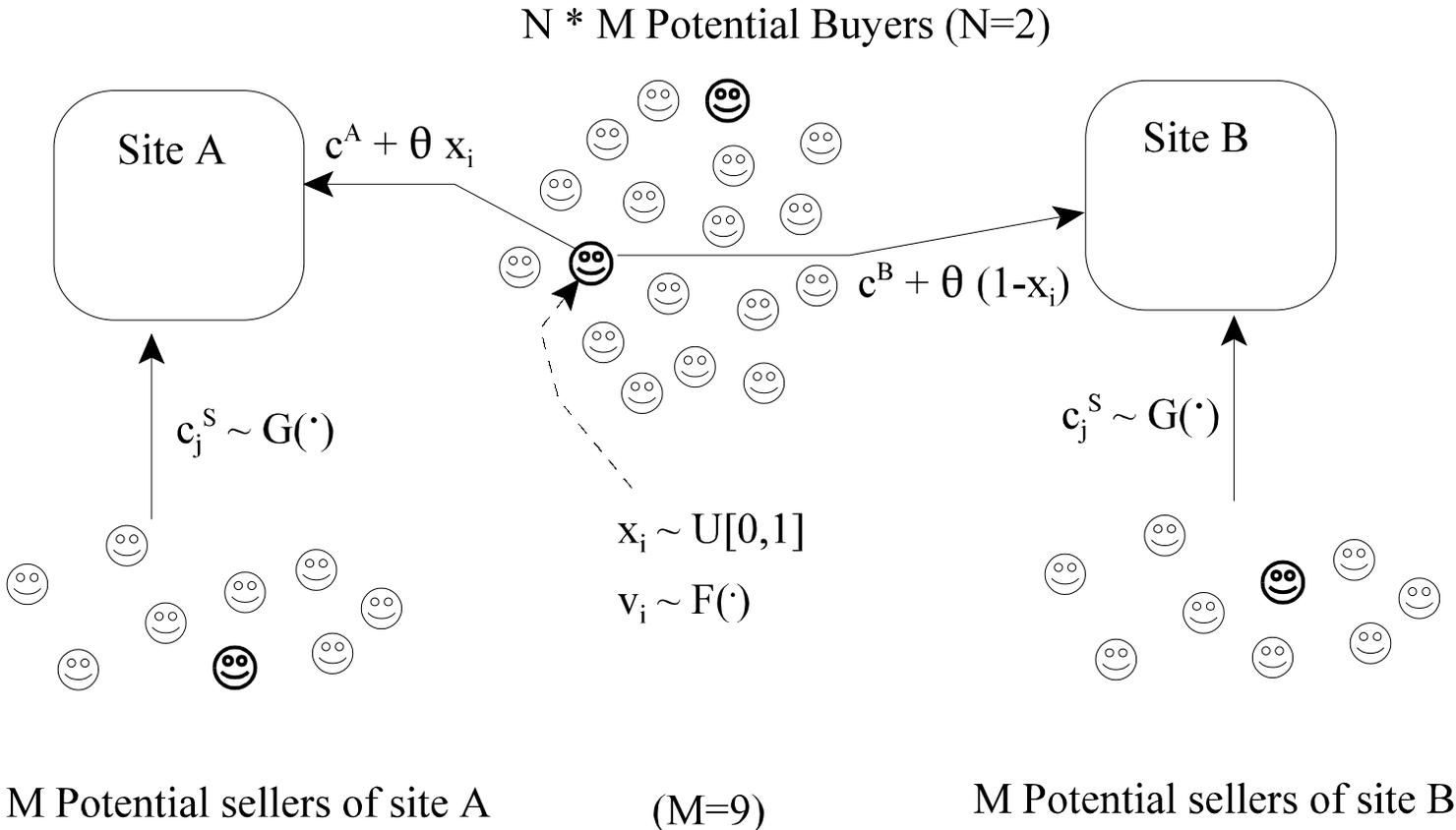
SITES AND BUYERS



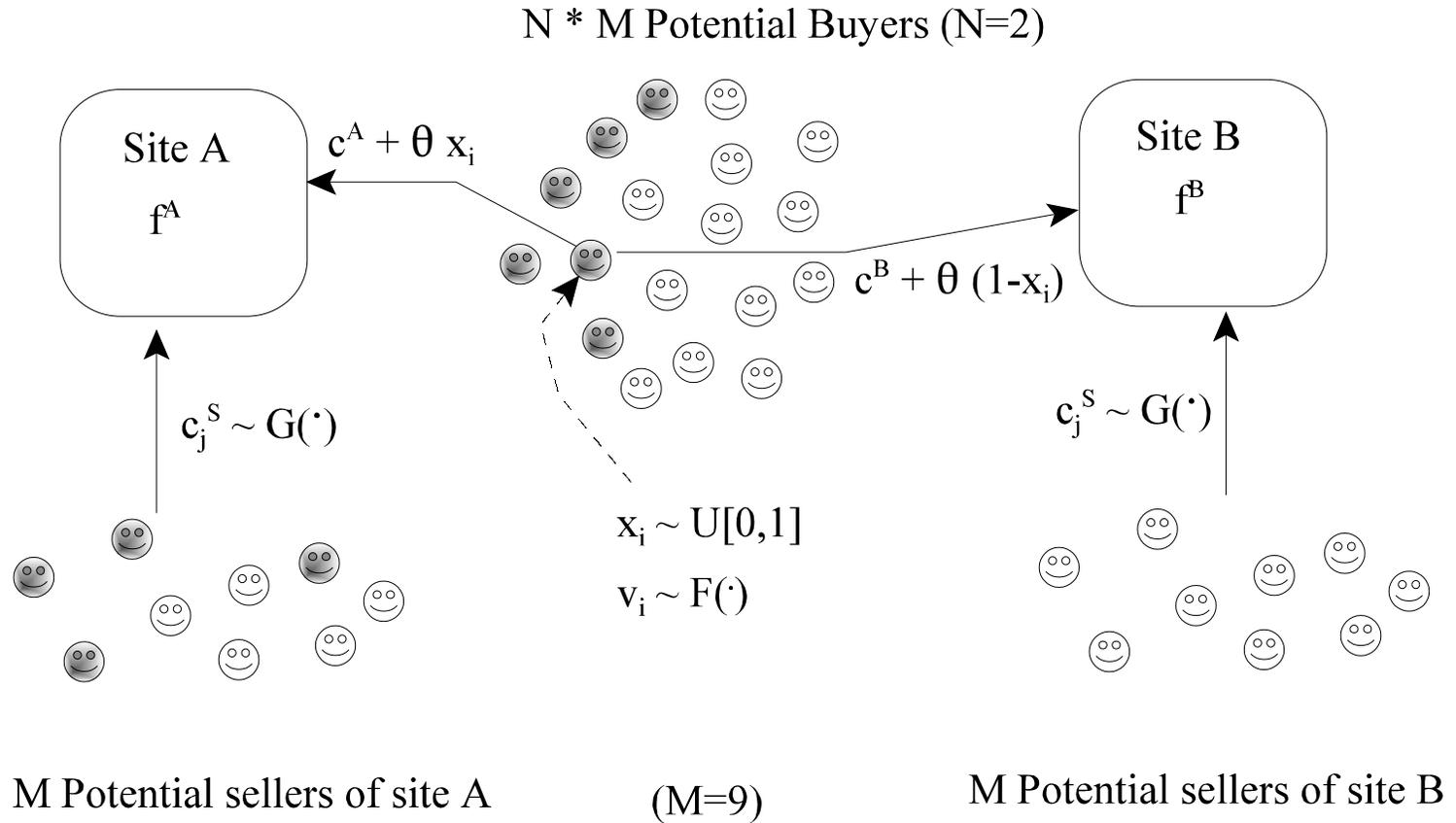
BUYER AND SELLER INTERACTIONS



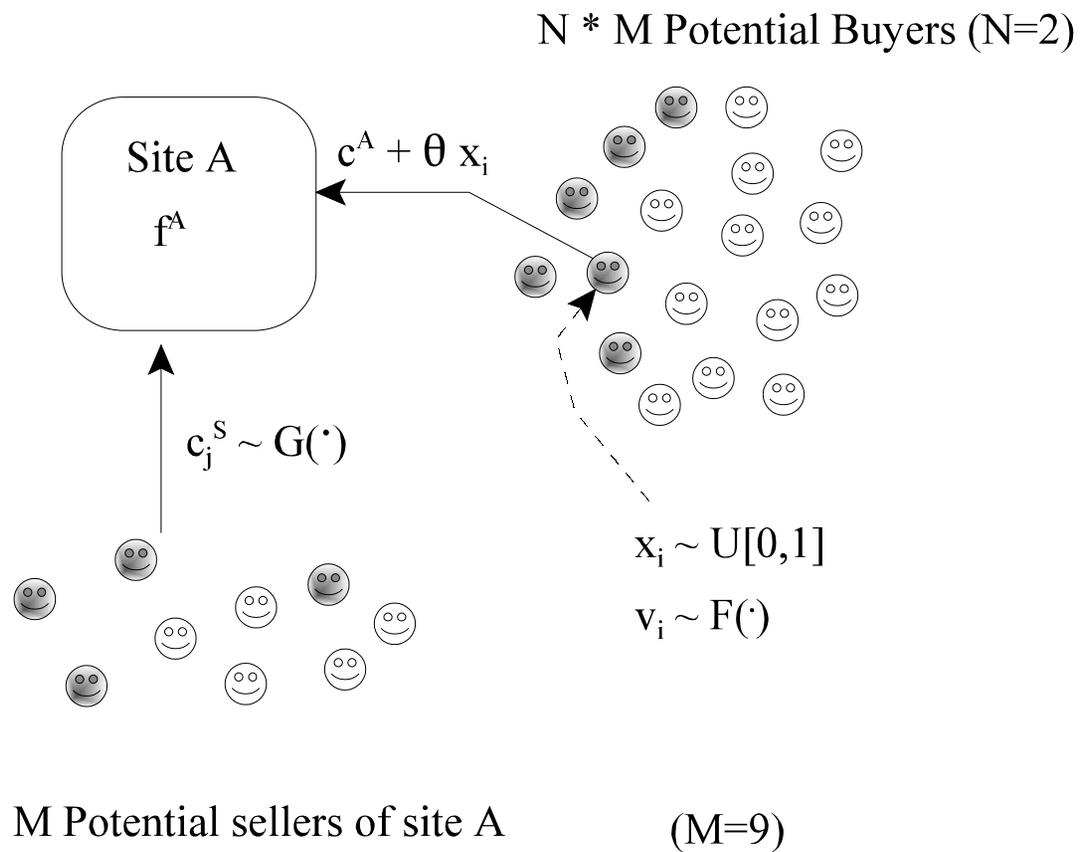
BUYER VS BUYER COMPETITION



HOSTING SITE PRICING AND EQUILIBRIUM



EQUILIBRIUM WITH A SINGLE (OR SINGLE ACTIVE) AUCTION HOSTING SITE



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GAME'S STRATEGIES/TIME-LINE

— Auction sites set fees.

— Sellers choose whether to offer their items in the auction hosting site, or keep them.

Each seller participates with probability q_A (or q_B) and sets the reserve for their item.

Seller actions based on (correct in equilibrium) beliefs about the number of consumers in site A (or B).

— Potential buyers choose whether to browse the site or watch t.v. (or choose between site A and site B). “Location” of indifferent buyer is x_c .

Valuations, v_i , and reserves are revealed to buyers only after browsing the site and looking at item's detailed information.

Buyer actions based on (correct in equilibrium) beliefs about q_A (or q_B).

POTENTIAL BUYER DECISION PROBLEM

Observation: We can look at the decisions of the potentially interacting groups buyers and sellers separately, ignoring the rest.

This is equivalent to fixing M to 1.

The variables q_A (and q_B) can thus be thought either as

- the probability of a particular seller participating in the website, or
- an index of the breadth of offerings and number of sellers in the website.

Define the payoff of a bidder with valuation v_i competing against $n-1$ other bidders in an auction with reserve r as

$$\Pi_{ExPost}^C(v, n, r) = \int_r^{v_{\max}} (n-1) f(z) F(v)^{n-2} (v-z) dz + F(r)^{n-1} (v-r)$$

The payoff of a bidder of “location” x_i who decides to browse site A is

$$\Pi_{Interim}^C(x_i, x_c, q_A | A) = \left[\sum_{n=1}^N H_A(n | x_c) \int_{r_A}^{v_{\max}} \Pi_{ExPost}^C(v, n, r_A) f(v) dv \right] q_A - \theta x_i - c^A$$

where

$H_A(n | x_c)$ = probability that there are n bidders in site A given that the cutoff location is x_c

Similarly, the bidder’s payoff from browsing site B is

$$\Pi_{Interim}^C(x_i, x_c, q_B | B) = \left[\sum_{n=1}^N H_B(n | x_c) \int_{r_B}^{v_{\max}} \Pi_{ExPost}^C(v, n, r_B) f(v) dv \right] q_B - \theta (1-x_i) - c^B$$

His payoff from browsing neither site is zero.

The potential buyer takes the decision that maximizes his payoff.

In a monopoly (or effective monopoly), x_c satisfies

$$\Pi_{Interim}^C(x_c, x_c, q_A | A) = 0$$

In other words, x_c can be thought of as being a function, $x_c(q_A)$, of the breadth of offerings, q_A , of site A.

In a duopoly, x_c satisfies

$$\Pi_{Interim}^C(x_c, x_c, q_A | A) = \Pi_{Interim}^C(x_c, x_c, q_B | B)$$

In other words, x_c can be thought of as being a function, $x_c(q_A, q_B)$, of the breadth of offerings, q_A and q_B , of both sites A and B.

POTENTIAL SELLER DECISION PROBLEM

A seller's revenue when he sets a reserve r and has n participating bidders is

$$R_{ExPost}^S(n, r) = \int_r^{v_{\max}} v n (n-1) f(v) F(v)^{n-2} [1 - F(v)] dv + r n F(r)^{n-1} [1 - F(r)]$$

The payoff of a site A potential seller (gross of the entry cost) payoff when

- The probabilities of seller participation (or breadth of offerings) is q_A and q_B , and
- The site fee is f^A

is given by:

$$R_{Interim}^S(q_A, q_B, f^A | A) = \sum_{n=1}^N H_A(n | x_c(q_A, q_B)) R_{ExPost}^S(n, r_A) - f^A$$

We often refer to this expression as the “Revenue Function.”

The corresponding revenue function for site B is

$$R_{Interim}^S(q_A, q_B, f^B | B) = \sum_{n=1}^N H_B(n | x_c(q_A, q_B)) R_{ExPost}^S(n, r_B) - f^B$$

When site A is a monopoly, x_c depends only on q_A and we thus have:

$$R_{Interim}^S(q_A, f^A | A) = \sum_{n=1}^N H_A(n | x_c(q_A)) R_{ExPost}^S(n, r_A) - f^A$$

The reserve is assumed to be

- i. Set by the seller to maximize his profits, and
- ii. Unobserved by the buyers until after they commit to a site.

A seller participates in a site if his revenue exceeds his entry costs.

ENTRY EQUILIBRIUM

The entry costs of a seller whose cost realization is in the q^{th} quantile is

$$C(q) = G^{-1}(q)$$

This is the inverse cost distribution function (or simply “cost function”).

An *Entry Equilibrium* is a value of q_A such that

$$R_{Interim}^S(q_A, f^A | A) - C(q_A) = 0$$

or, in the duopoly, a pair of values q_A and q_B such that

$$R_{Interim}^S(q_A, q_B, f^A | A) - C(q_A) = 0 \quad \text{and} \quad R_{Interim}^S(q_A, q_B, f^B | B) - C(q_B) = 0$$

AUCTION HOSTING SITE'S PROBLEM

Denote by $q_A^{eq}(f^A, f^B)$ and $q_B^{eq}(f^A, f^B)$ the entry equilibrium fraction of potential sellers of either site that choose to sell their stuff.

Auction hosting sites are assumed to have no variable costs.

Therefore, the objectives of the two sites are to maximize

$$\Pi^A(f^A | f^B) = f^A q_A^{eq}(f^A, f^B) \quad \text{and} \quad \Pi^B(f^B | f^A) = f^B q_B^{eq}(f^A, f^B)$$

Note: If site A is a monopoly, its objective function is $\Pi^A(f^A) = f^A q_A^{eq}(f^A)$.

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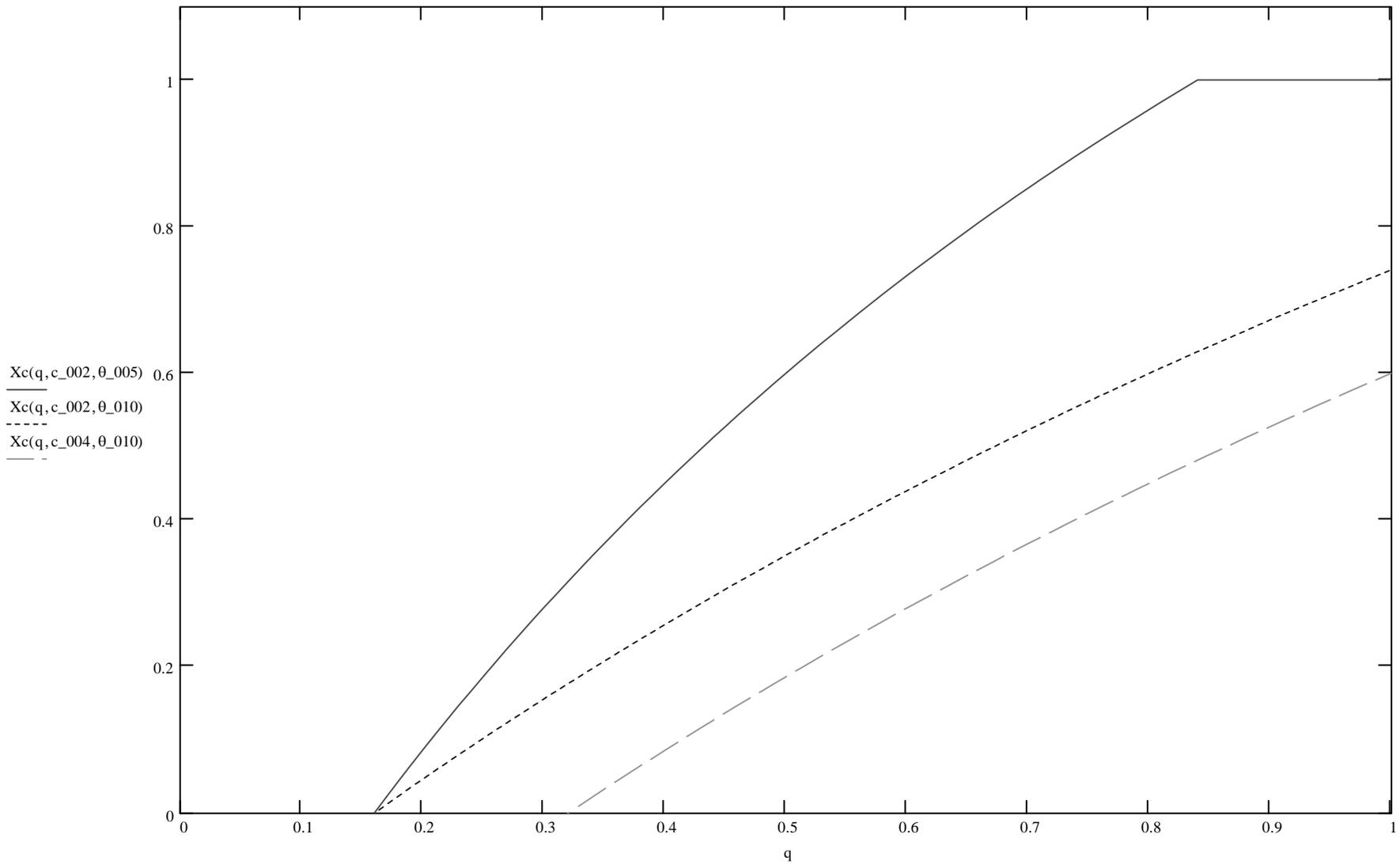


Figure 1. Location of critical consumer in monopoly.

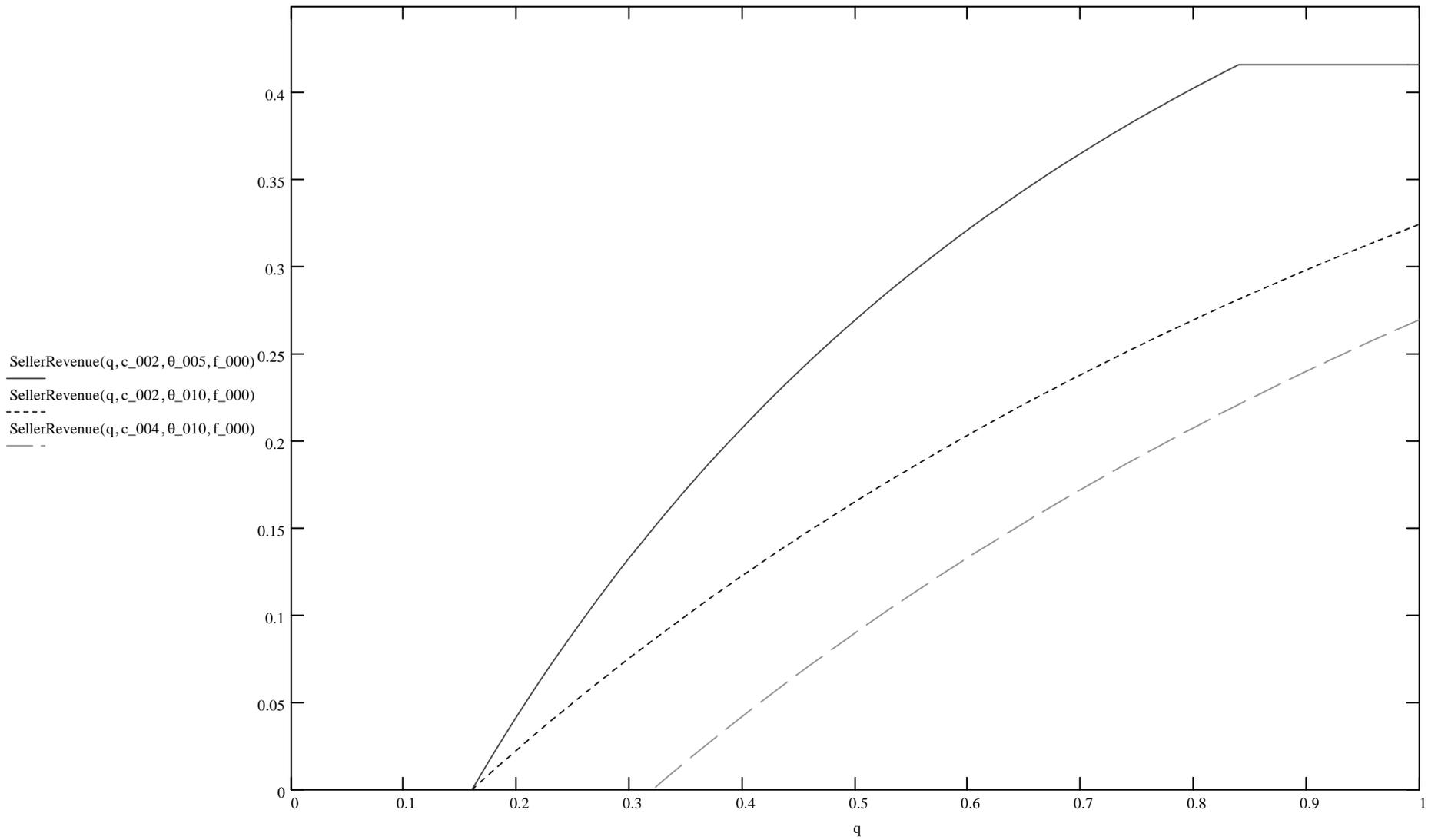


Figure 2. Seller revenue (gross of entry costs).

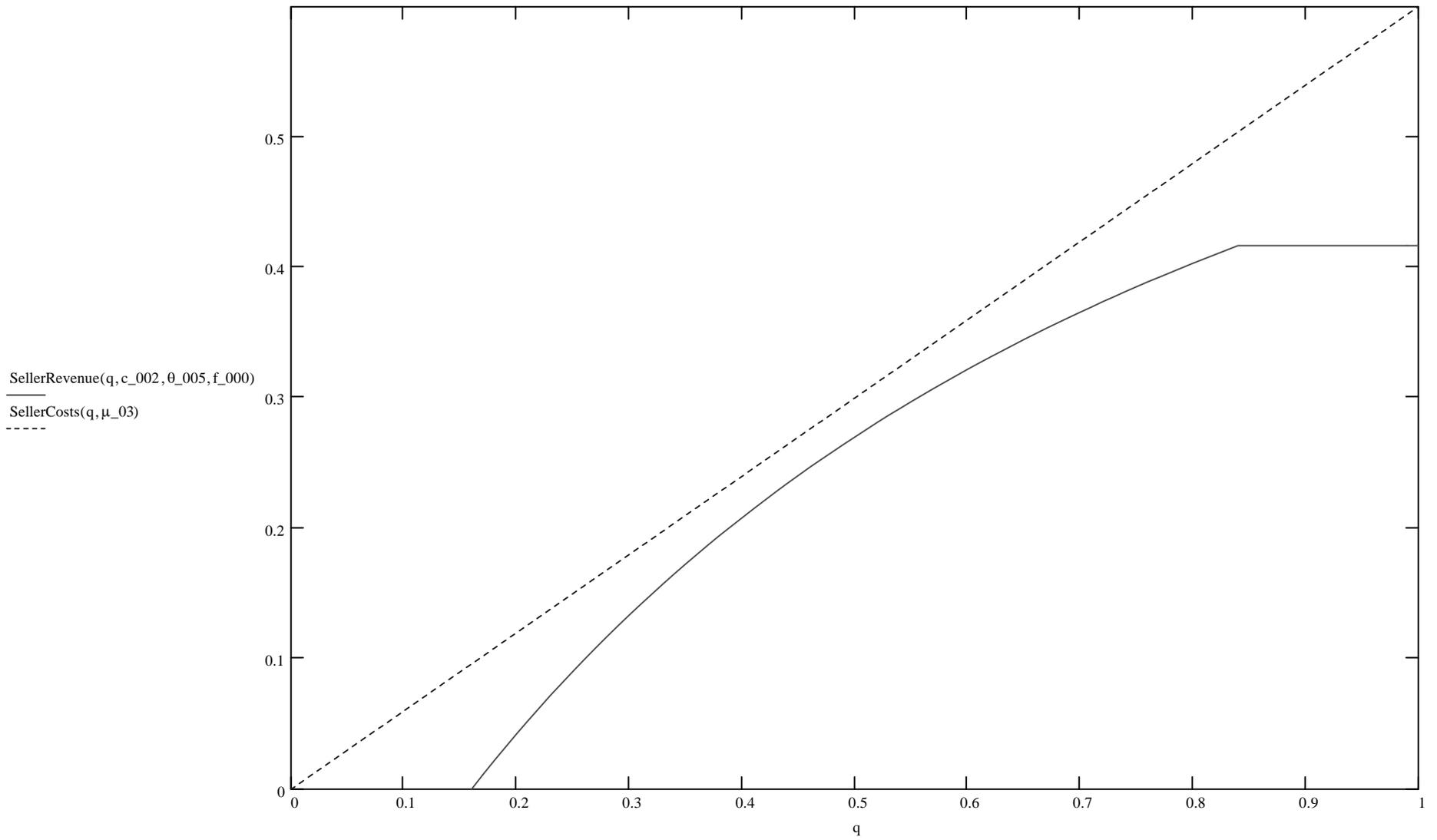


Figure 3. Market Equilibrium: Zero equilibrium supply (site shutdown).

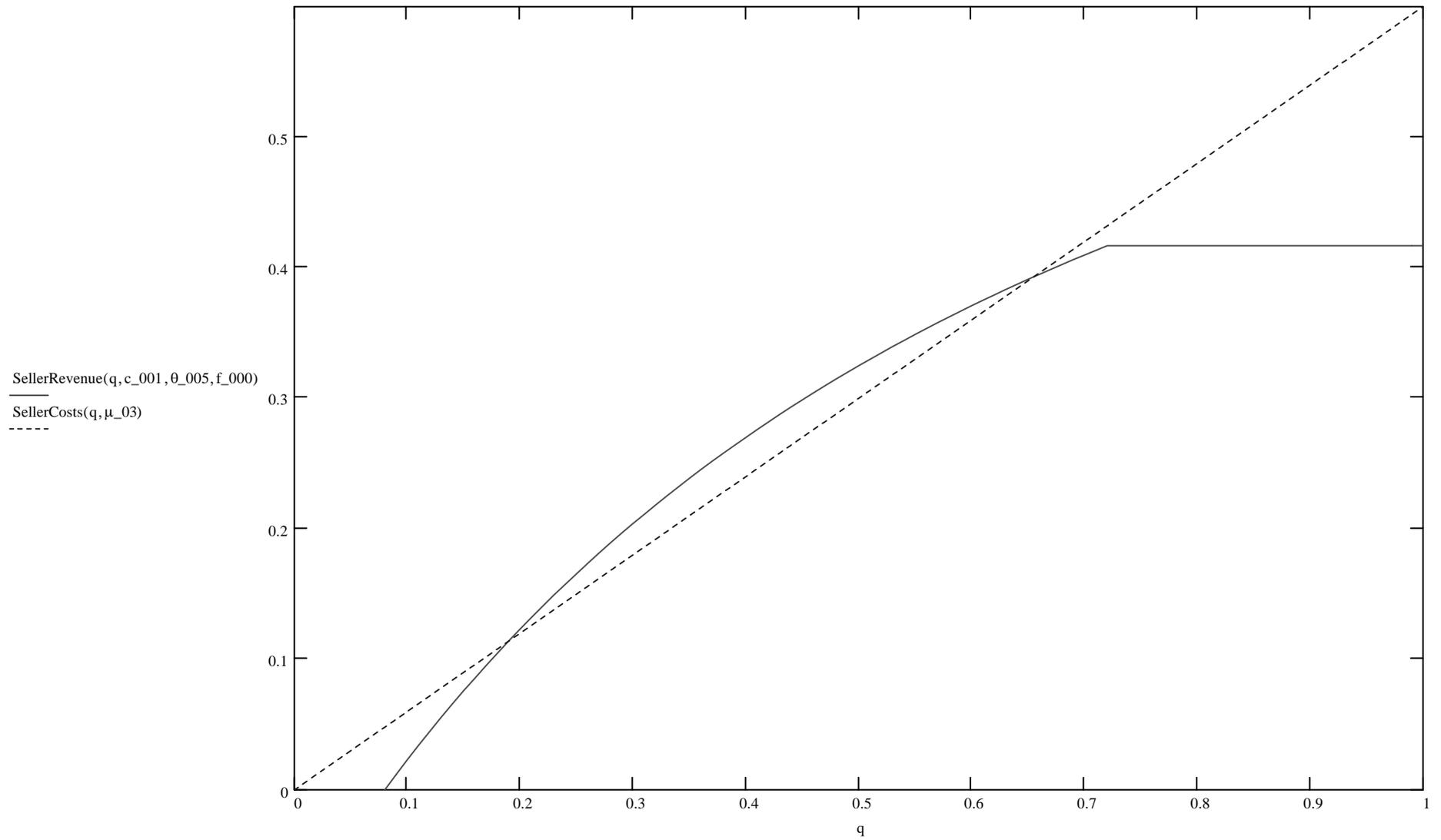


Figure 4. Market Equilibrium: Two interior entry equilibria (one stable).

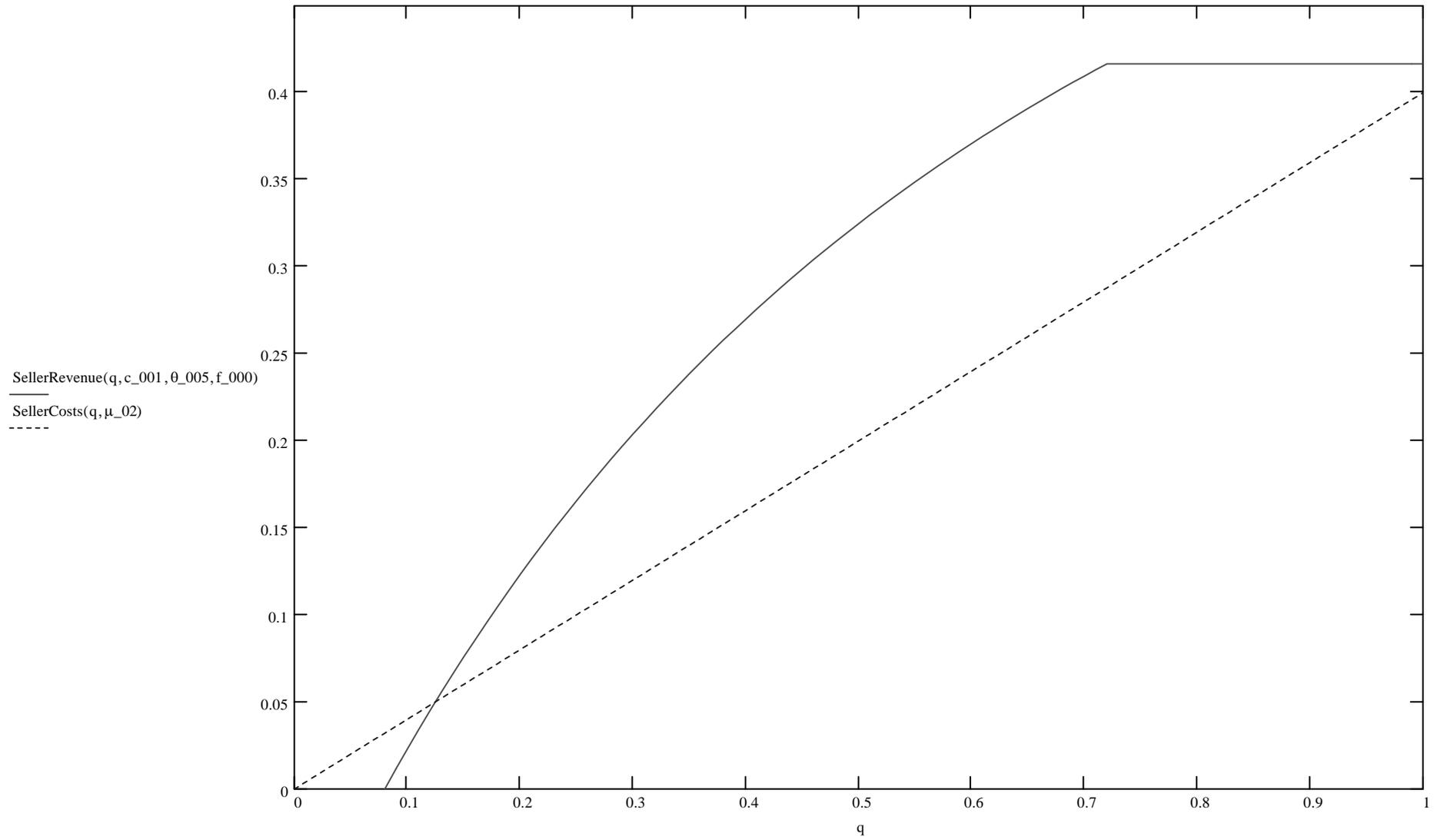


Figure 5. Market Equilibrium: One unstable interior entry equilibrium, stable full participation equilibrium.

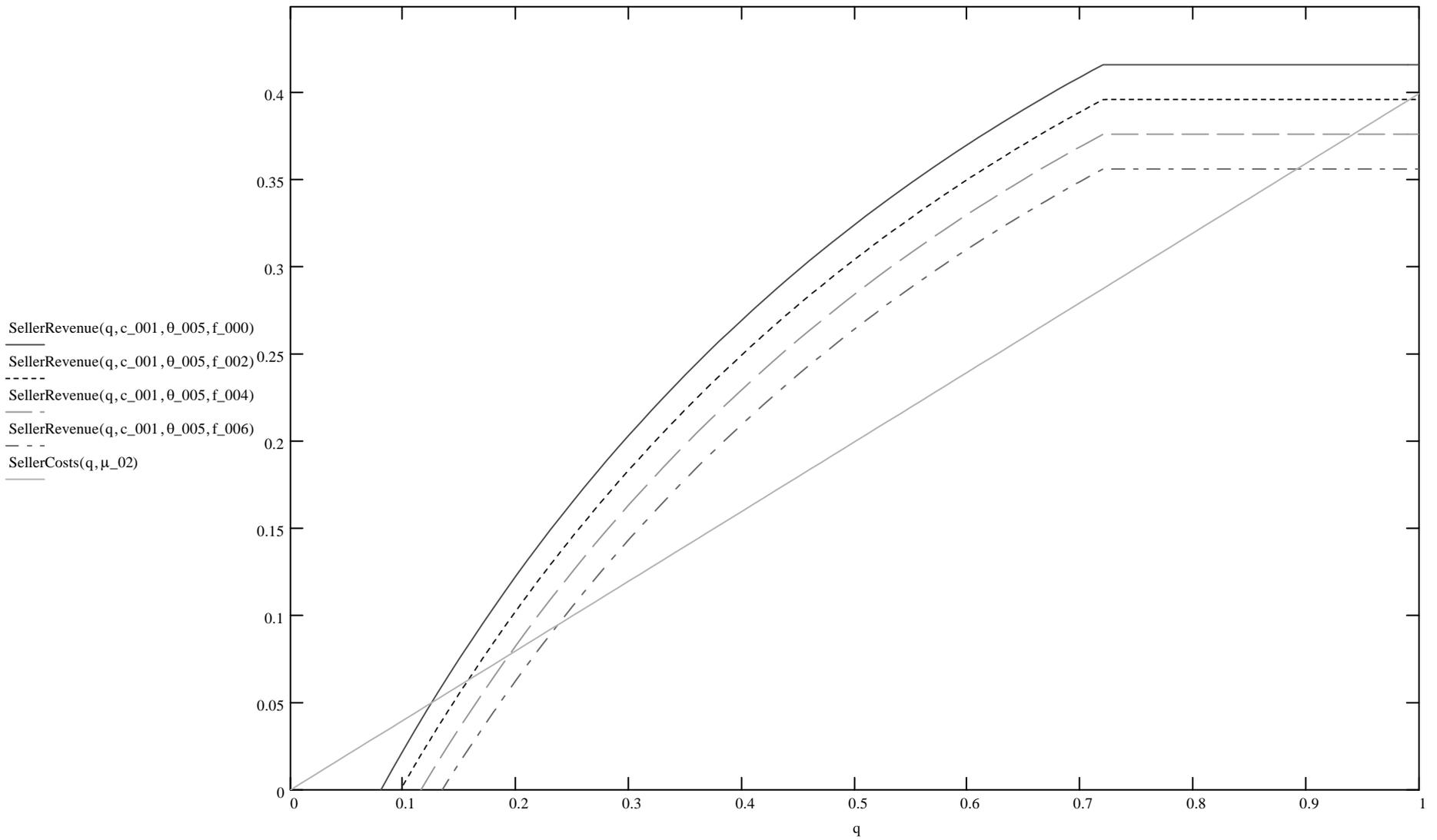


Figure 6. Market Equilibrium: Effect of an increase in listing fees.

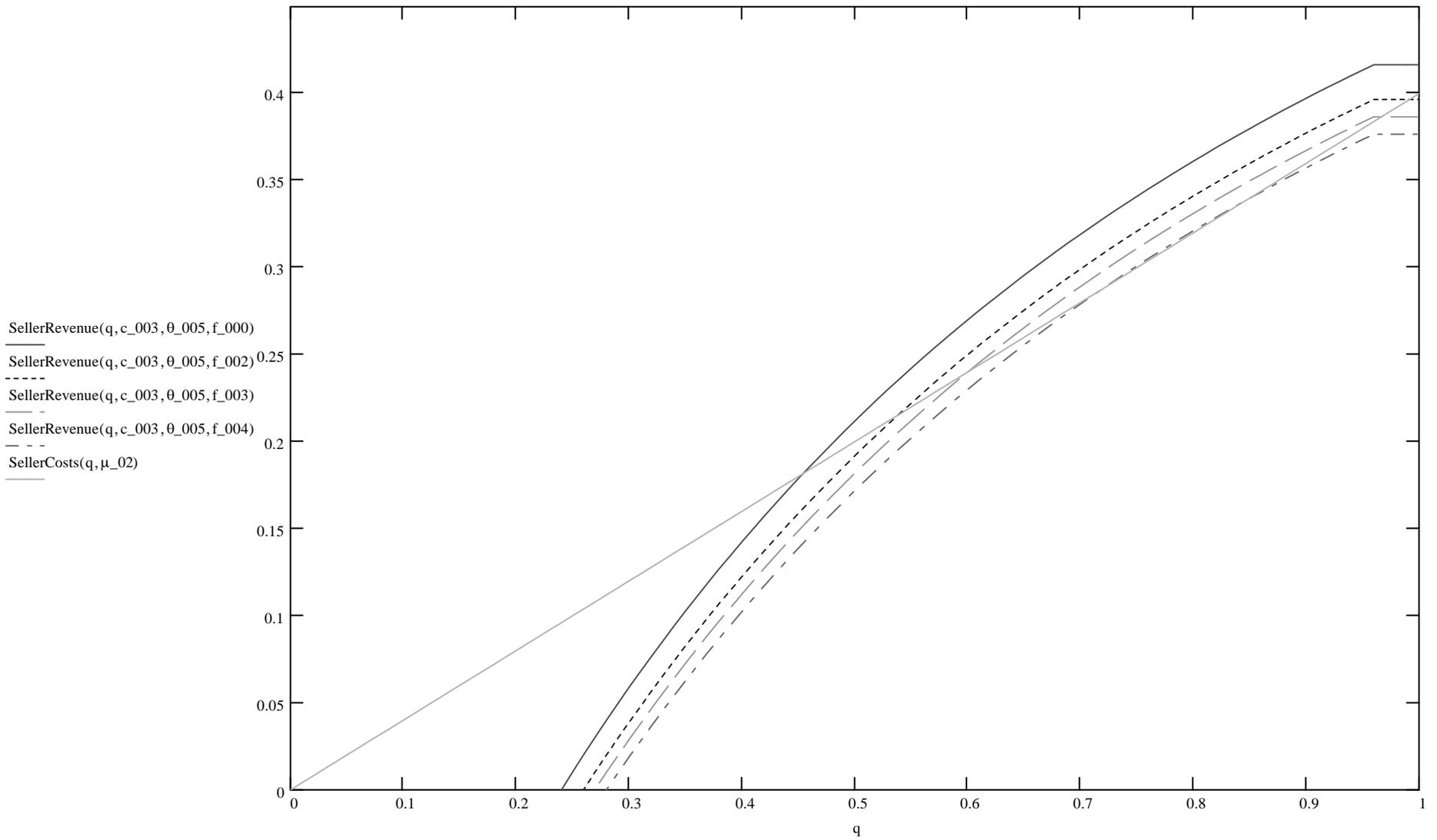


Figure 7. Market Equilibrium: Profitable increase in listing fees.

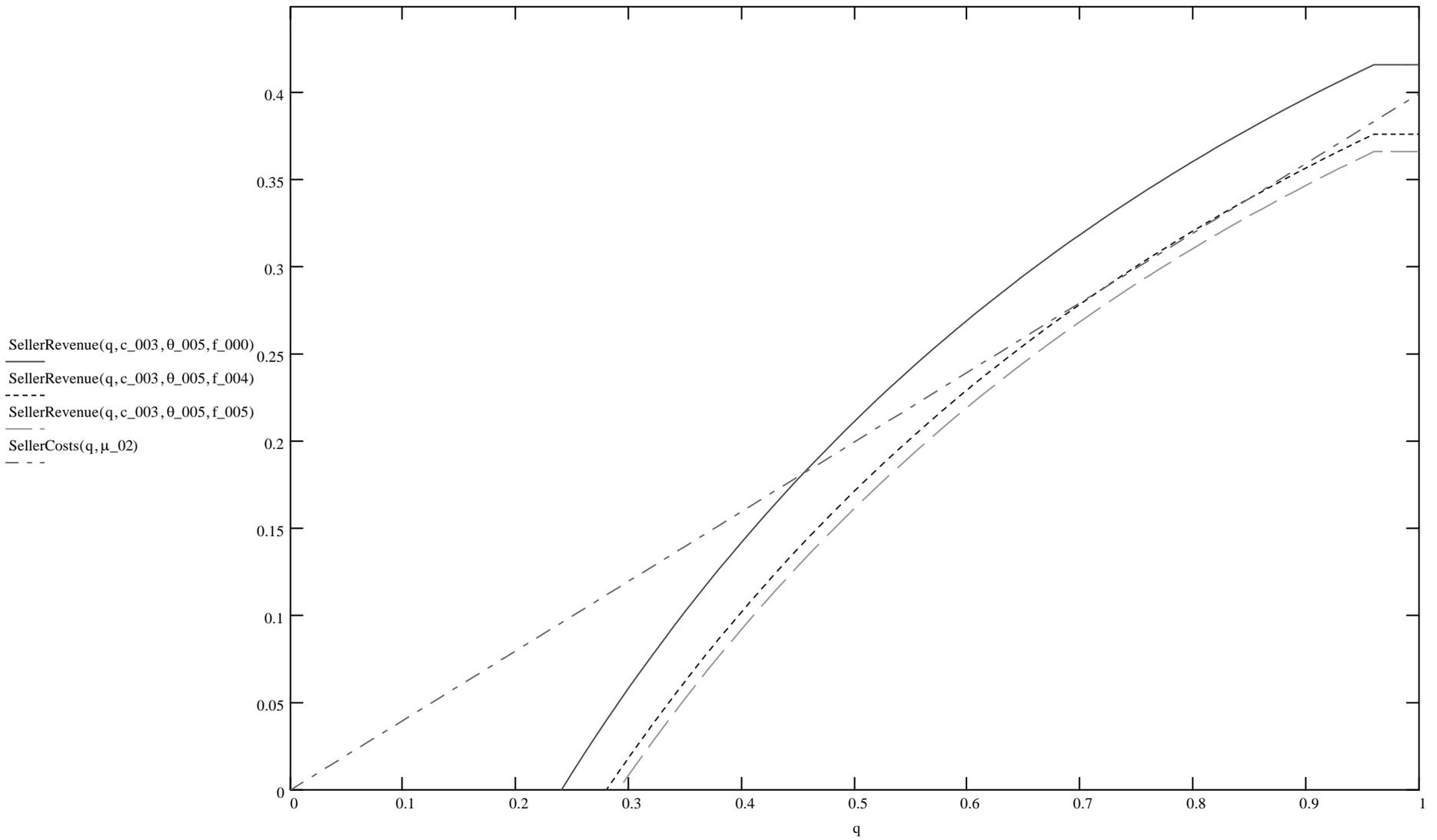


Figure 8. Market destroying effects of marginal changes in the listing fee.

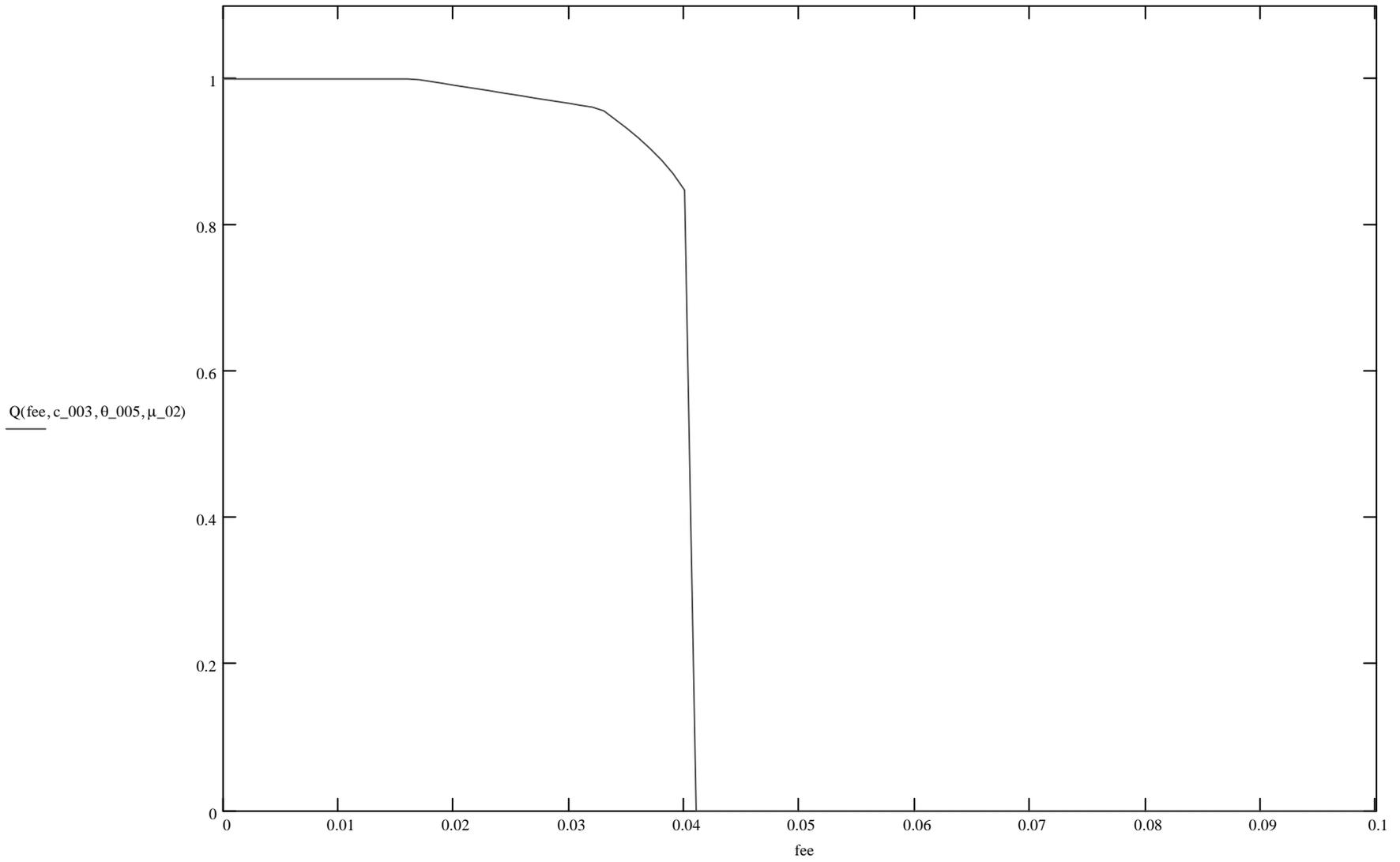


Figure 9. Auction hosting site demand curve.

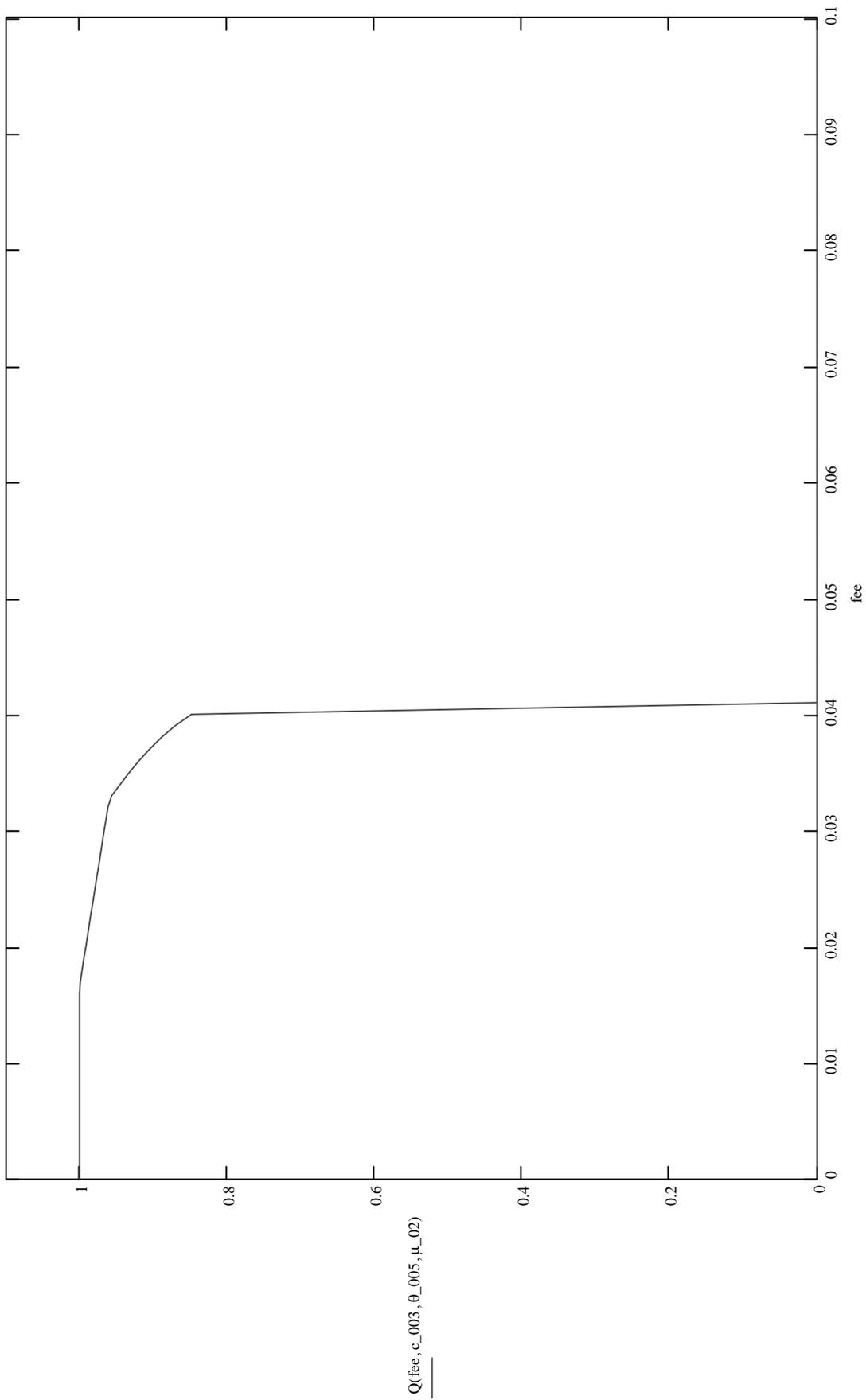


Figure 9. Auction hosting site demand curve.

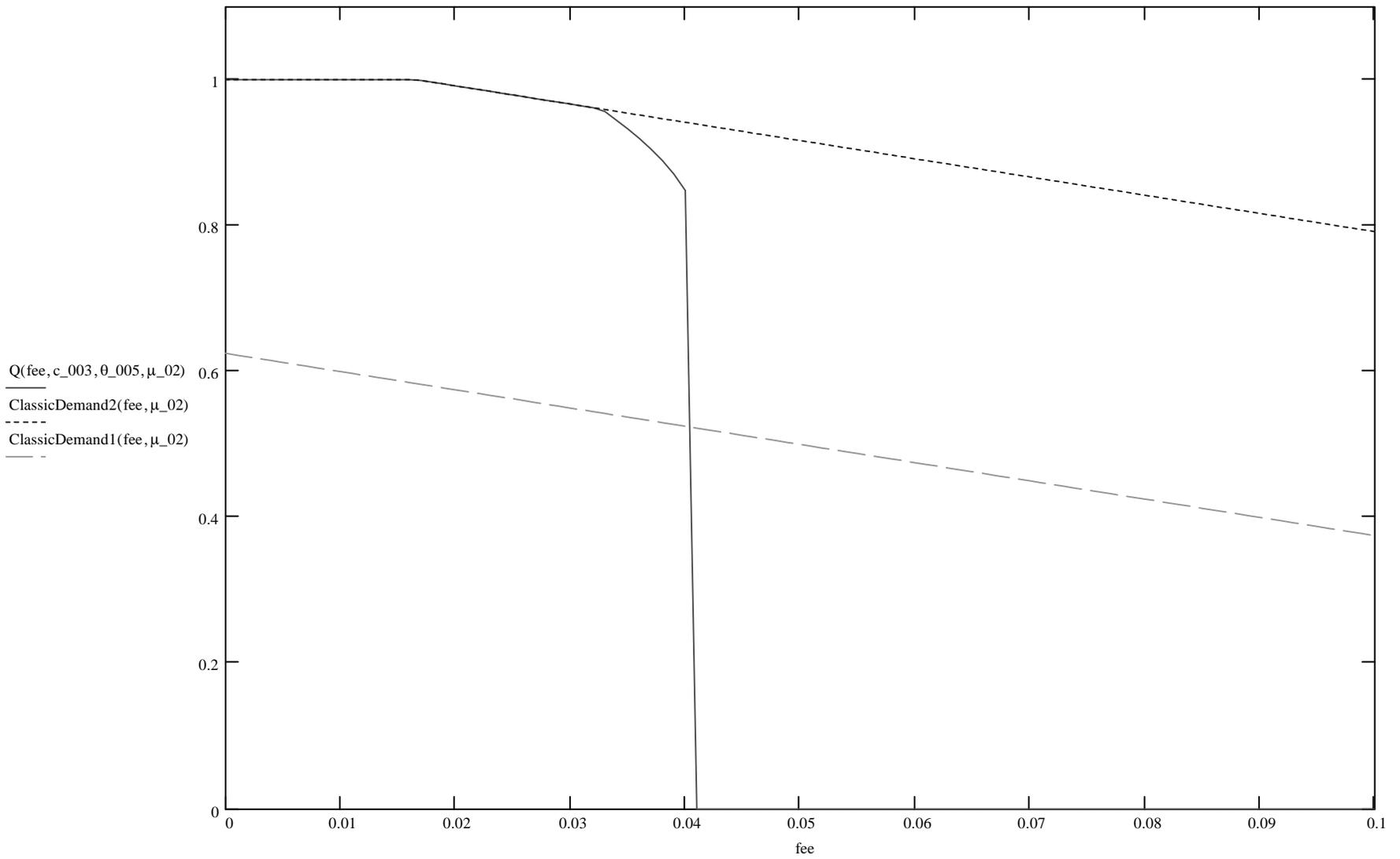


Figure 10. Comparison with "normal" (no feedback) auction site demand curves.

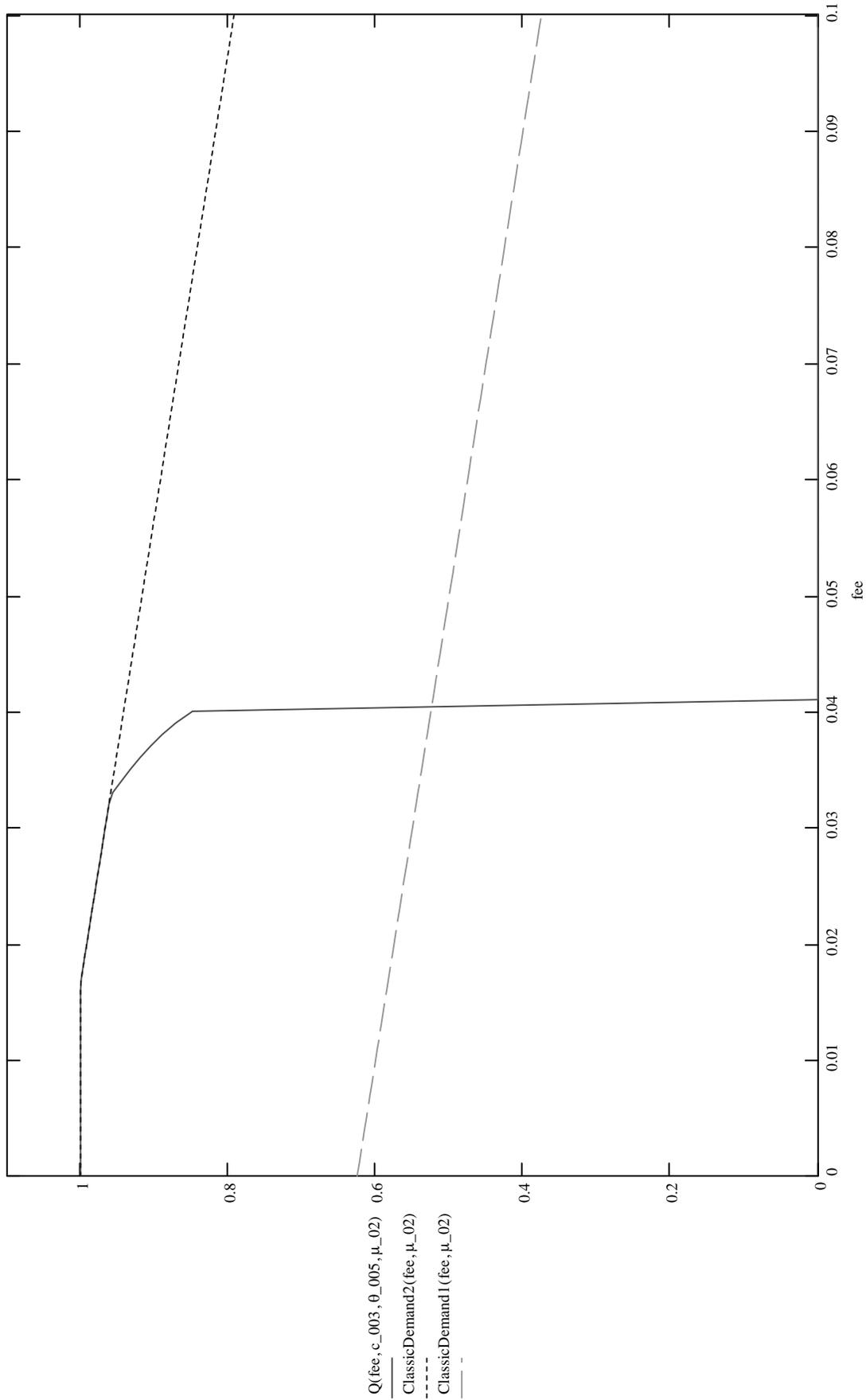


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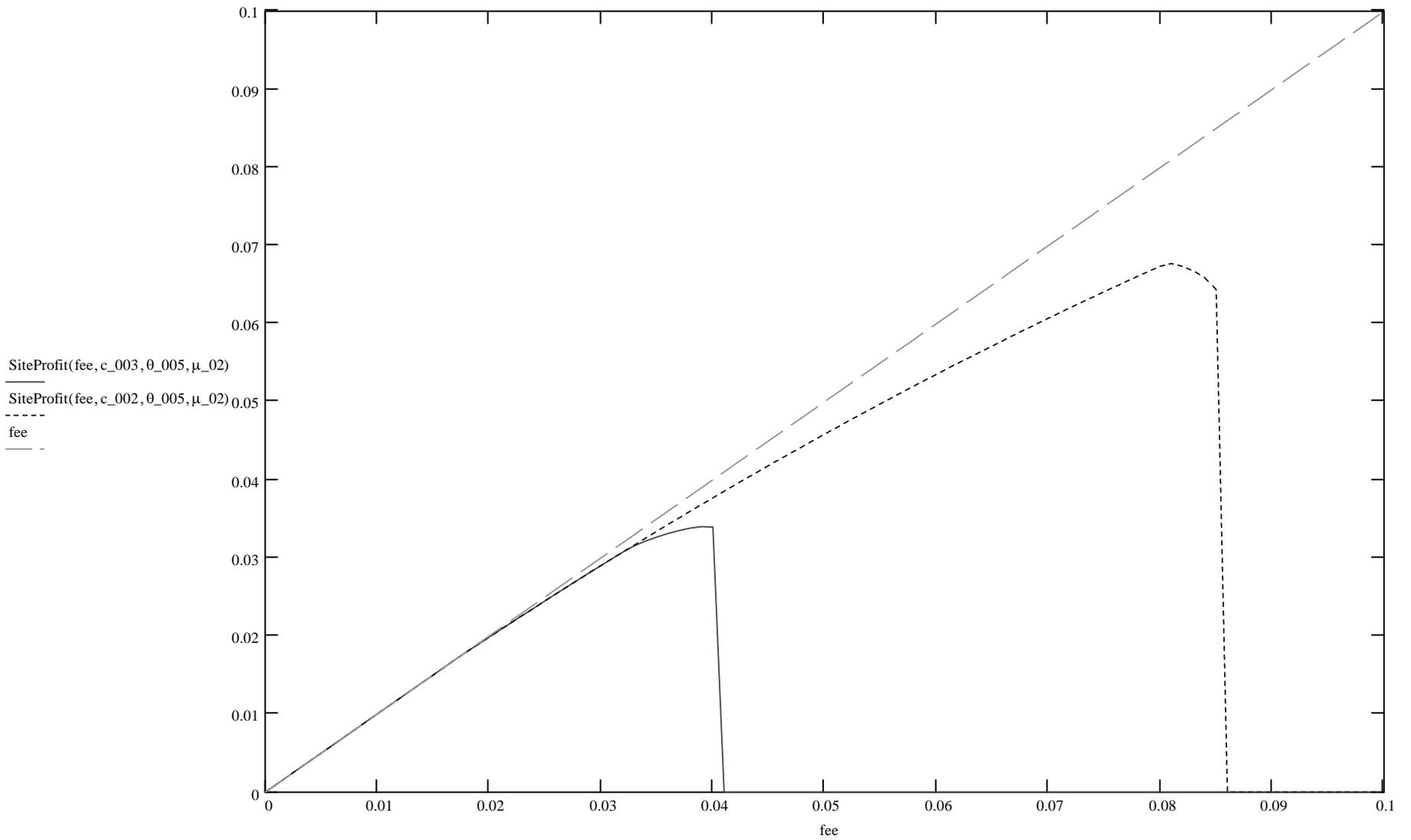


Figure 11. Site profit functions: changes in mean consumer attendance costs.

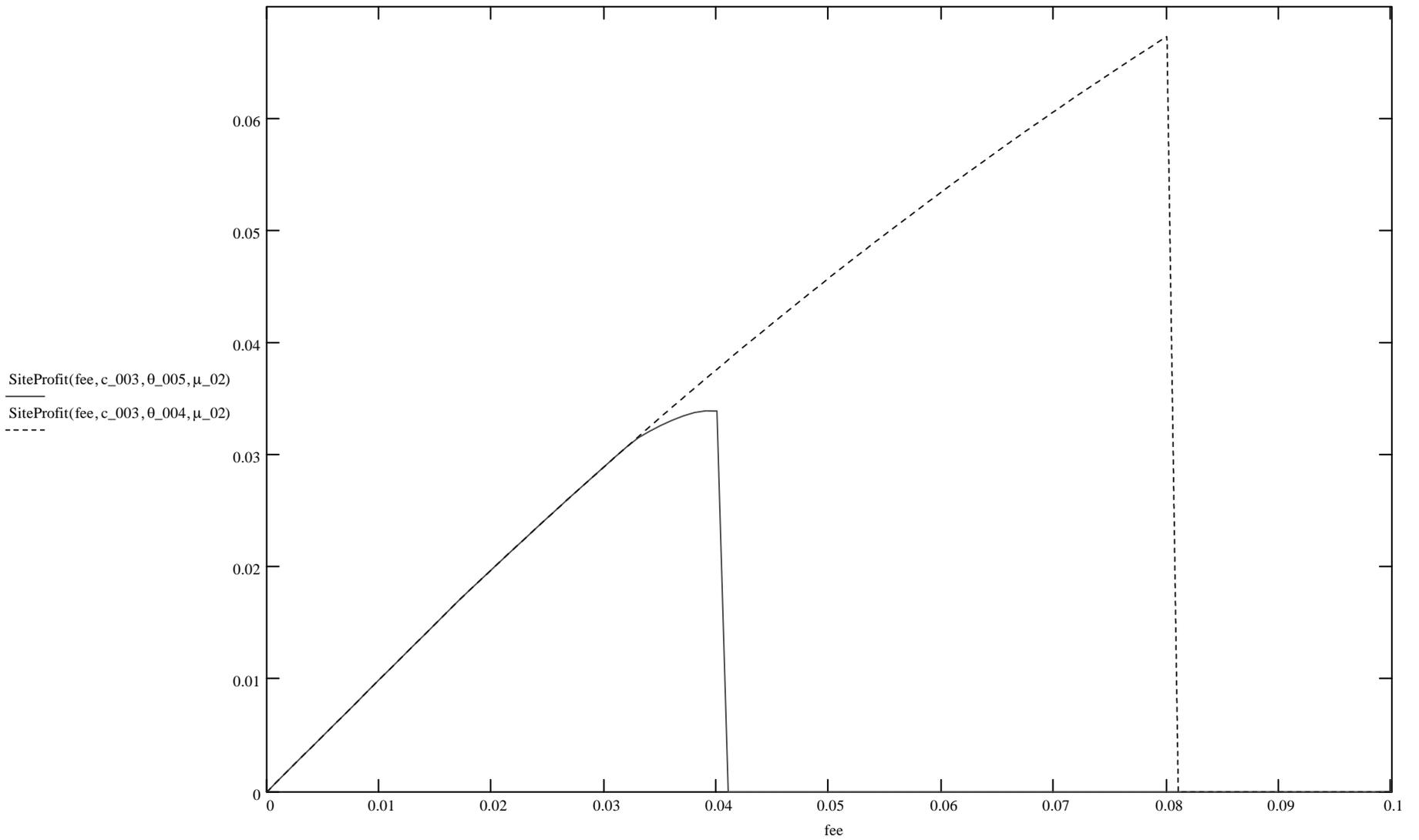


Figure 12. Site profit functions: changes in the dispersion of consumer attendance costs.

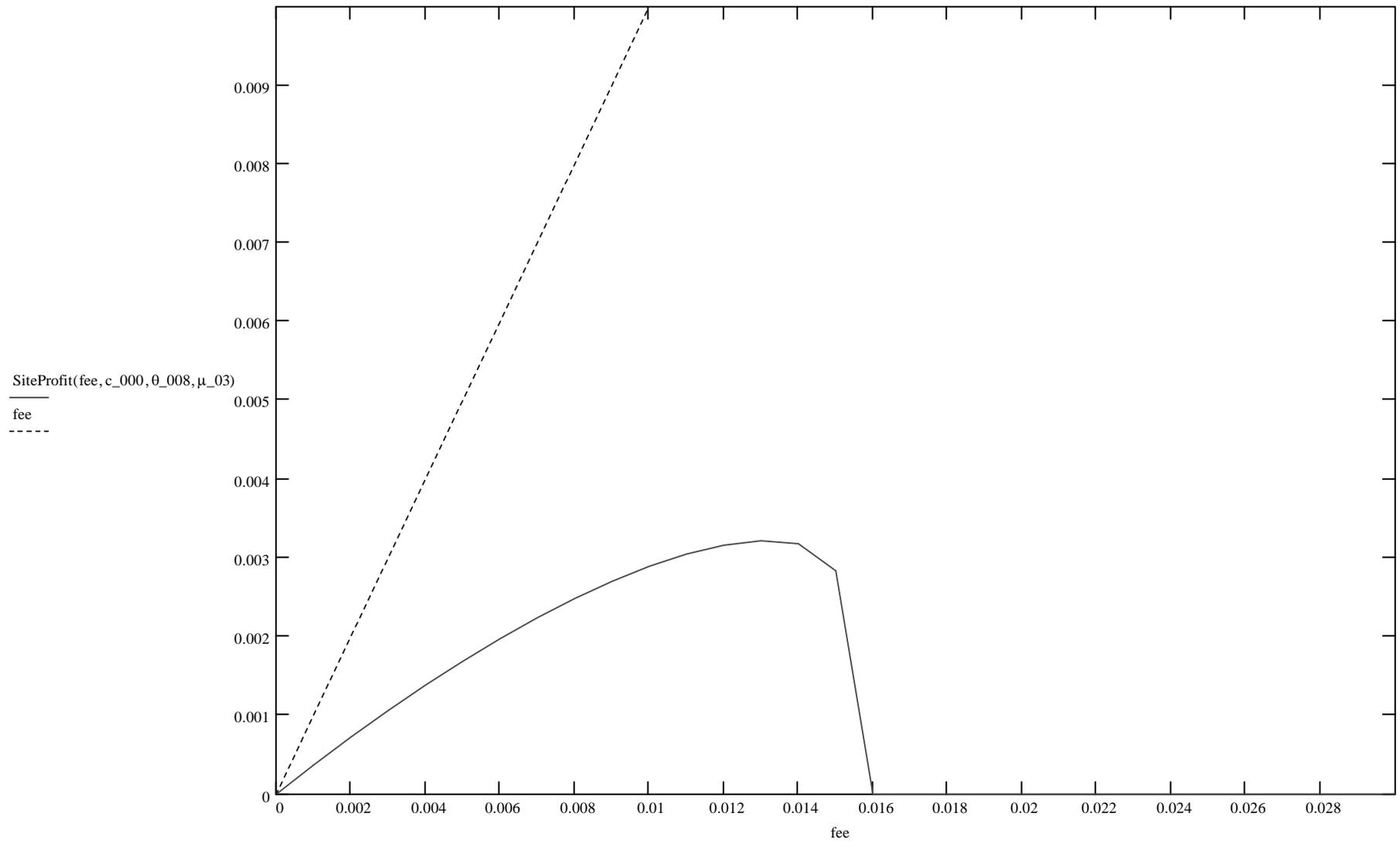


Figure 13. A site profit function with an interior optimum ($q < 100\%$, differentiable maximum).

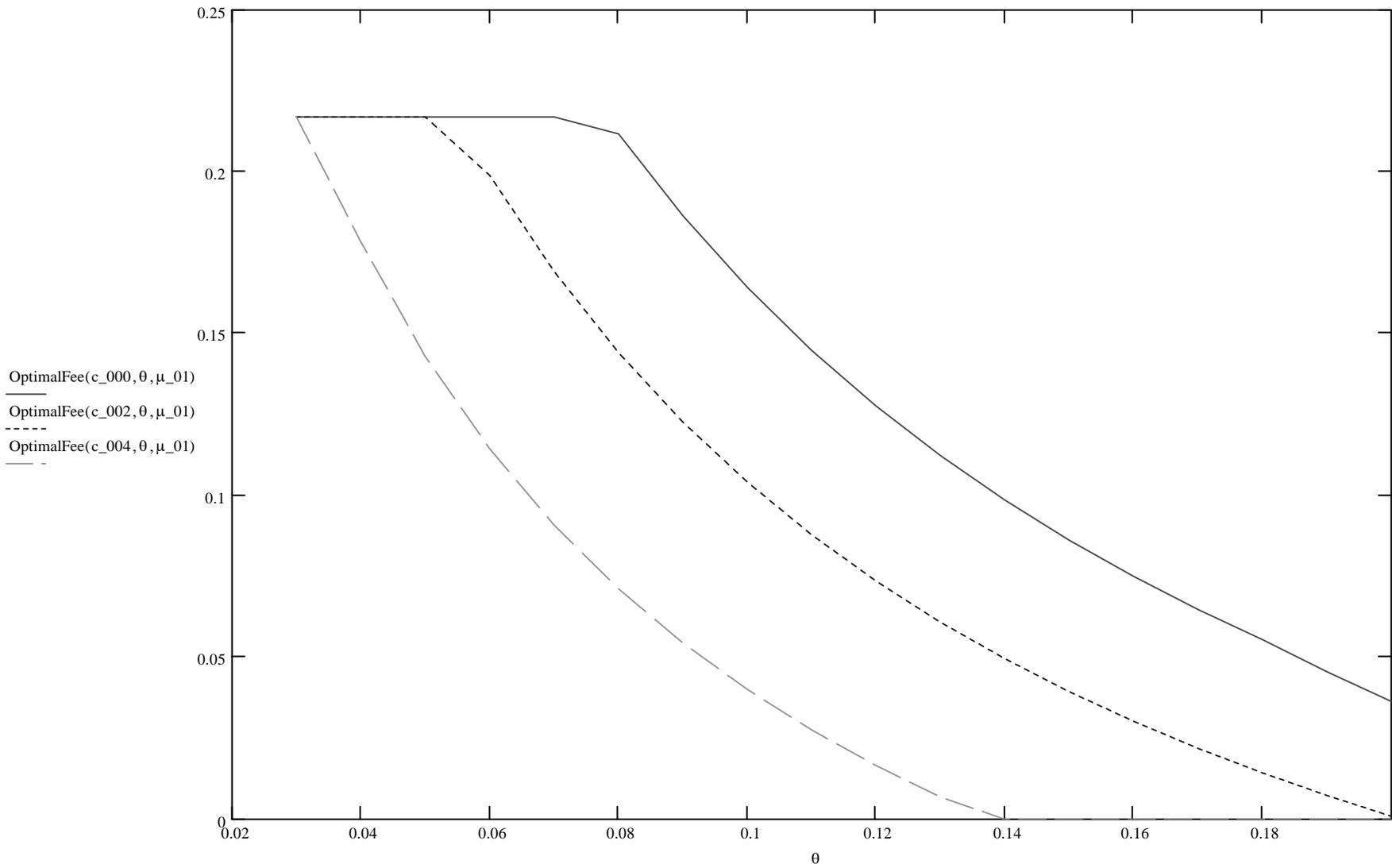


Figure 14. Optimal list fee as a function of the buyer participation costs.

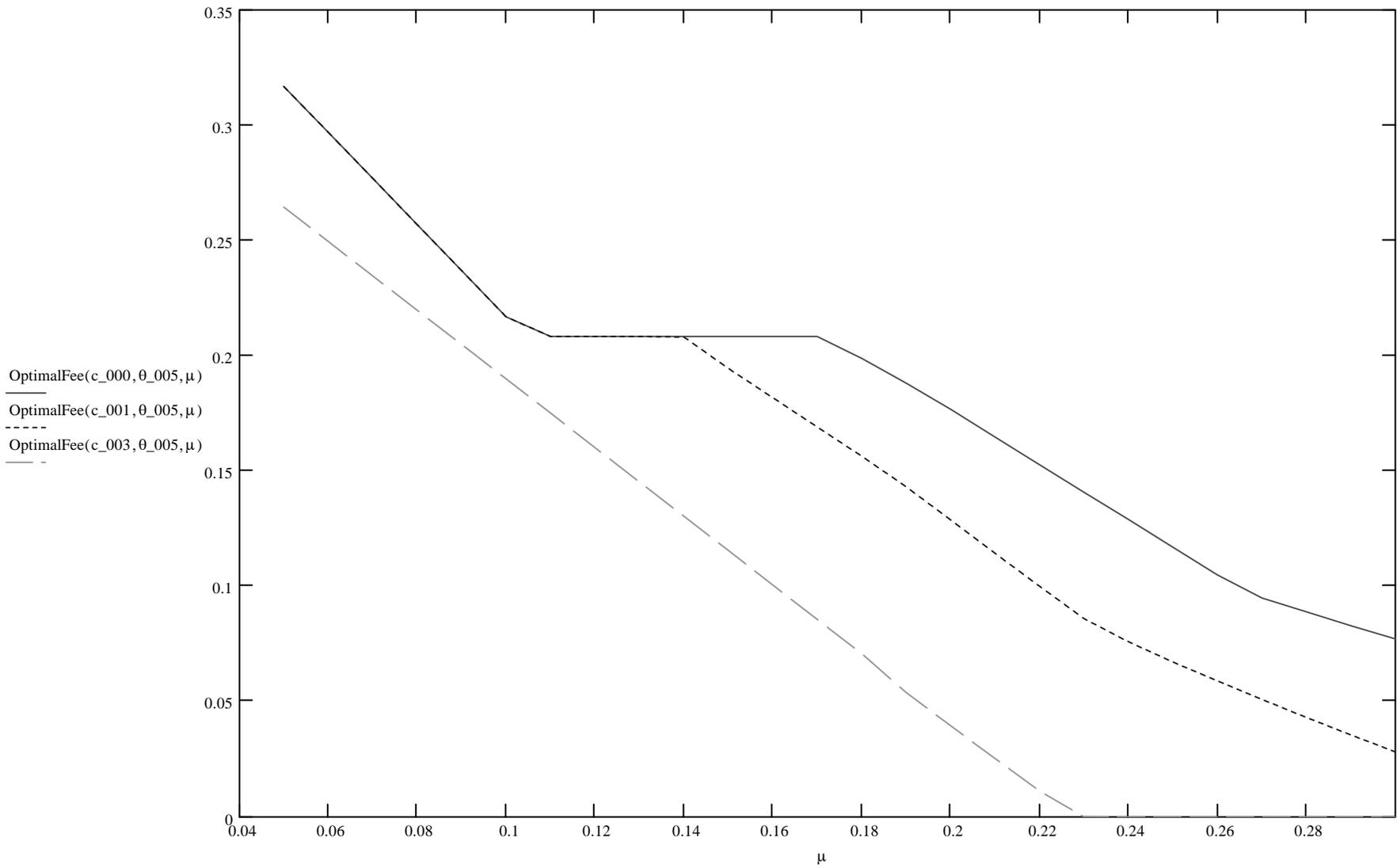


Figure 15. Optimal list fee as a function of the mean of seller participation costs.

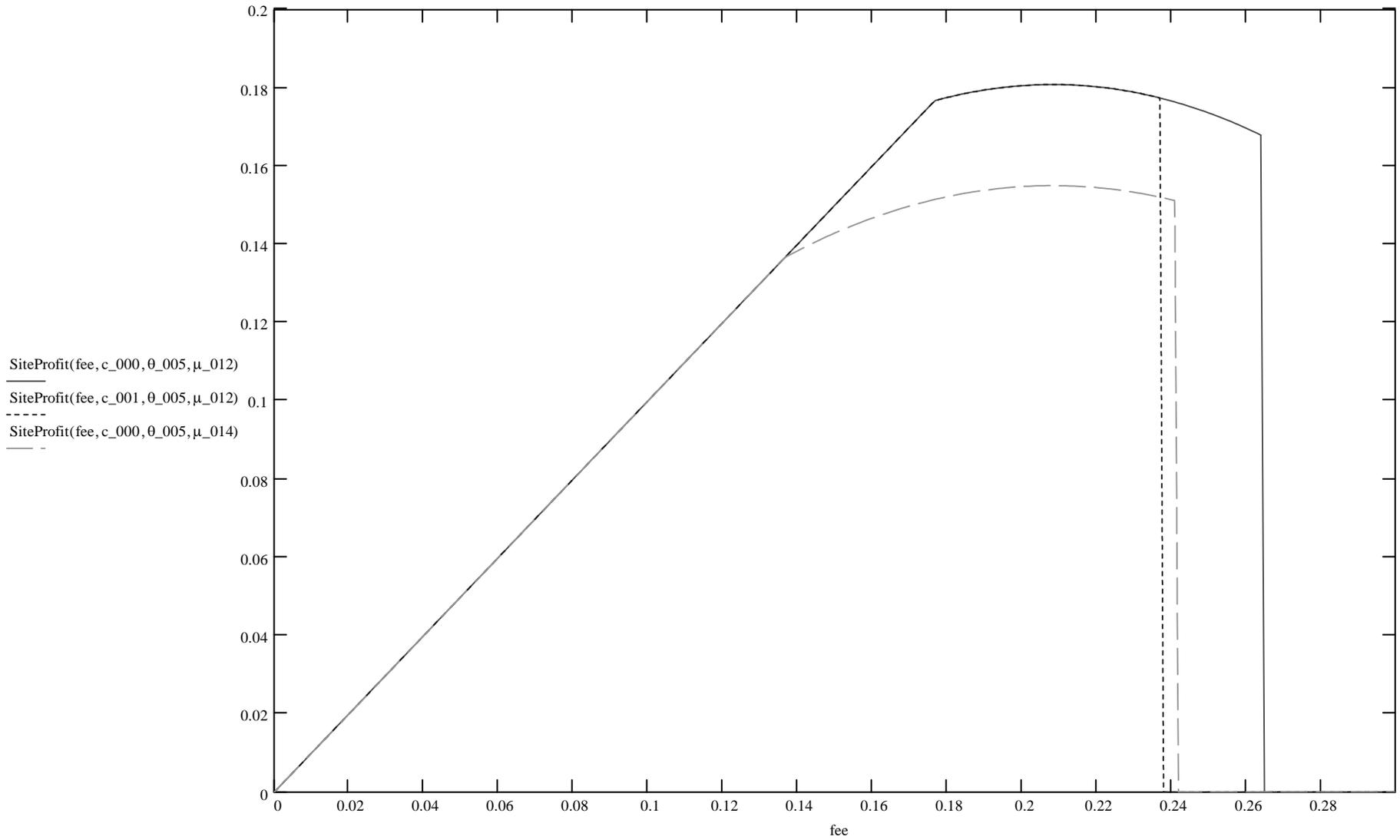


Figure 16. Possible local unresponsiveness of optimal fee to seller or buyer costs.

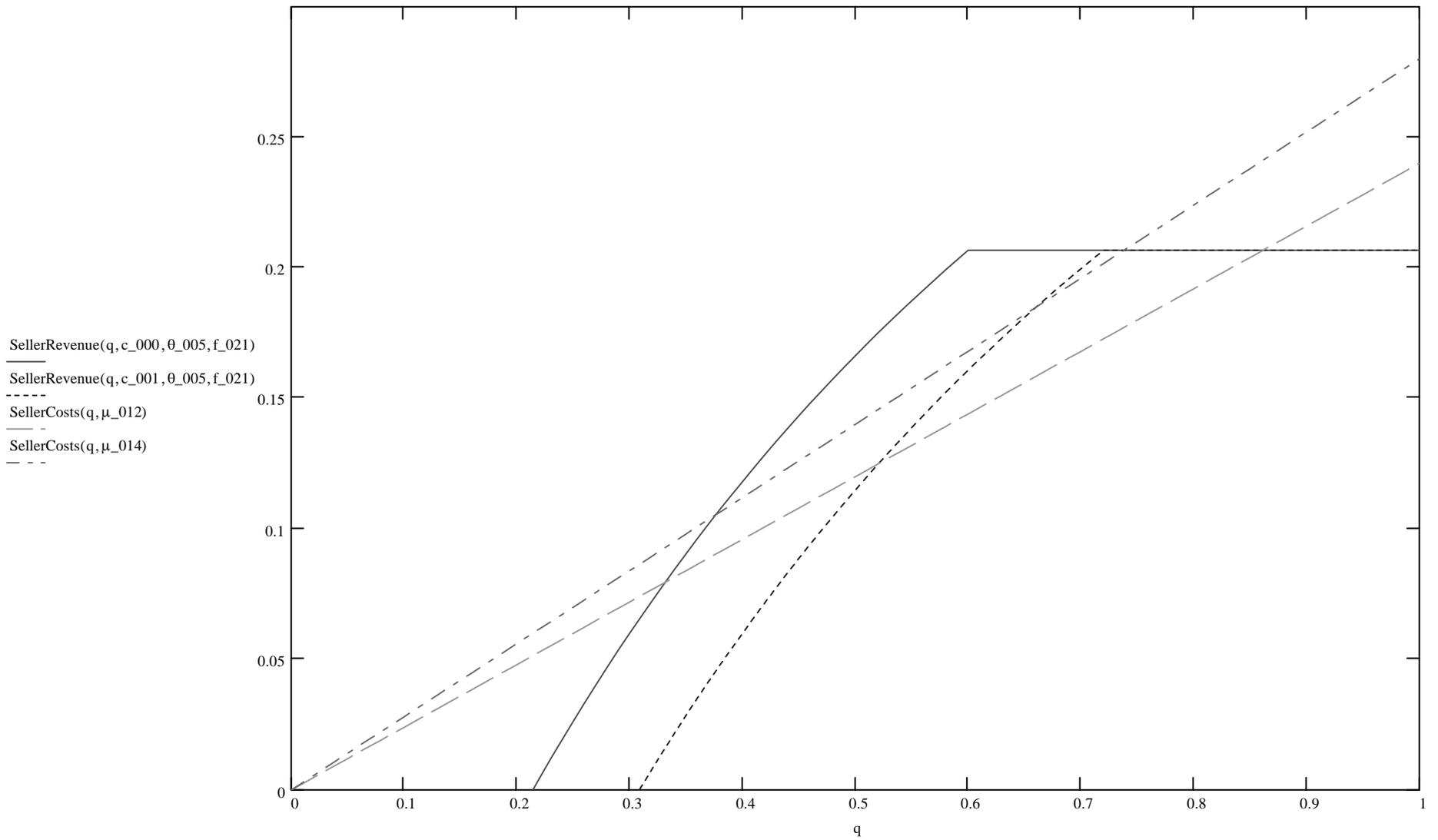


Figure 16b. A closer look at the local unresponsiveness of optimal fee to seller or buyer costs.

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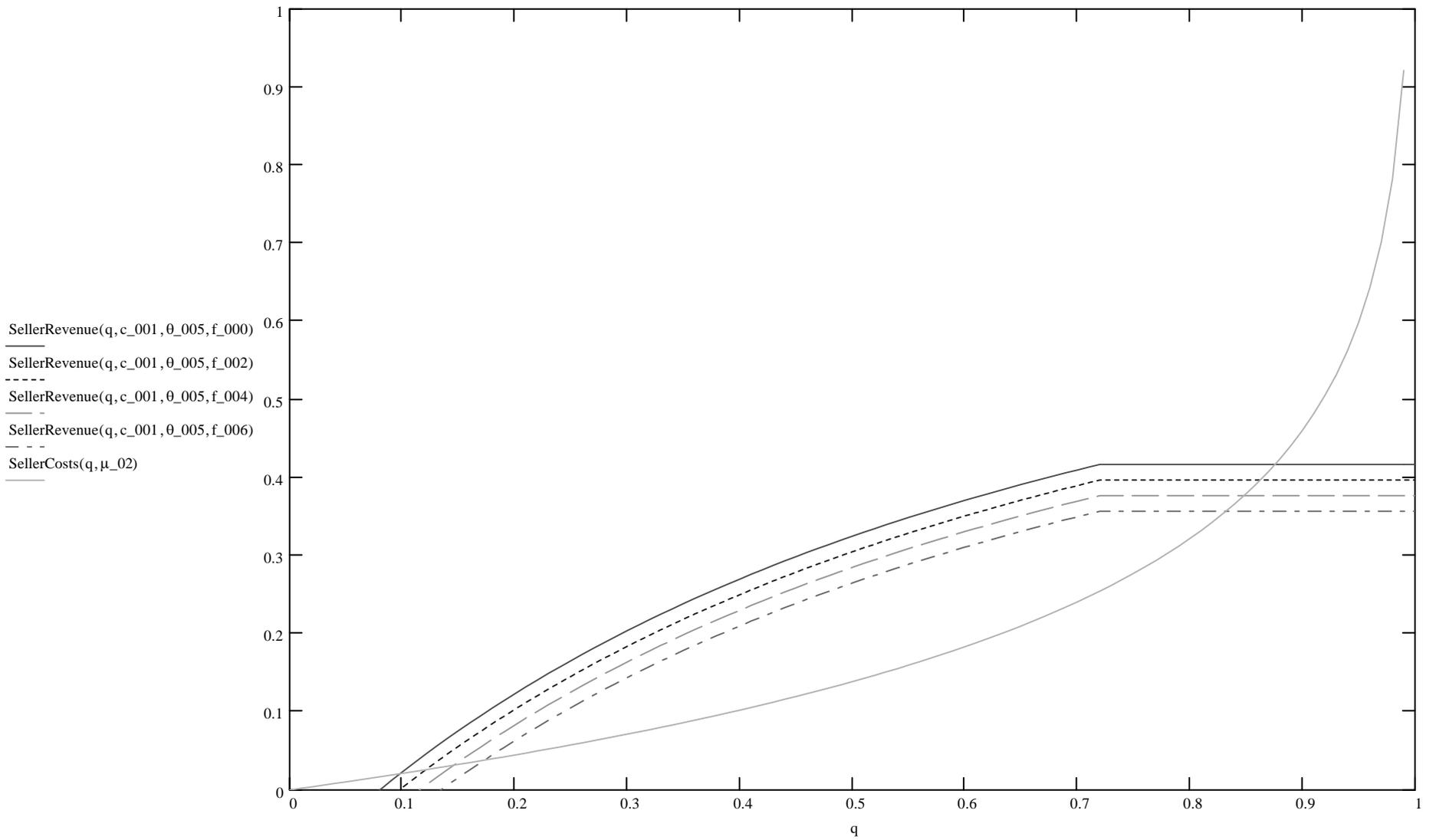


Figure 17. Market Equilibrium with exponentially distributed seller costs.

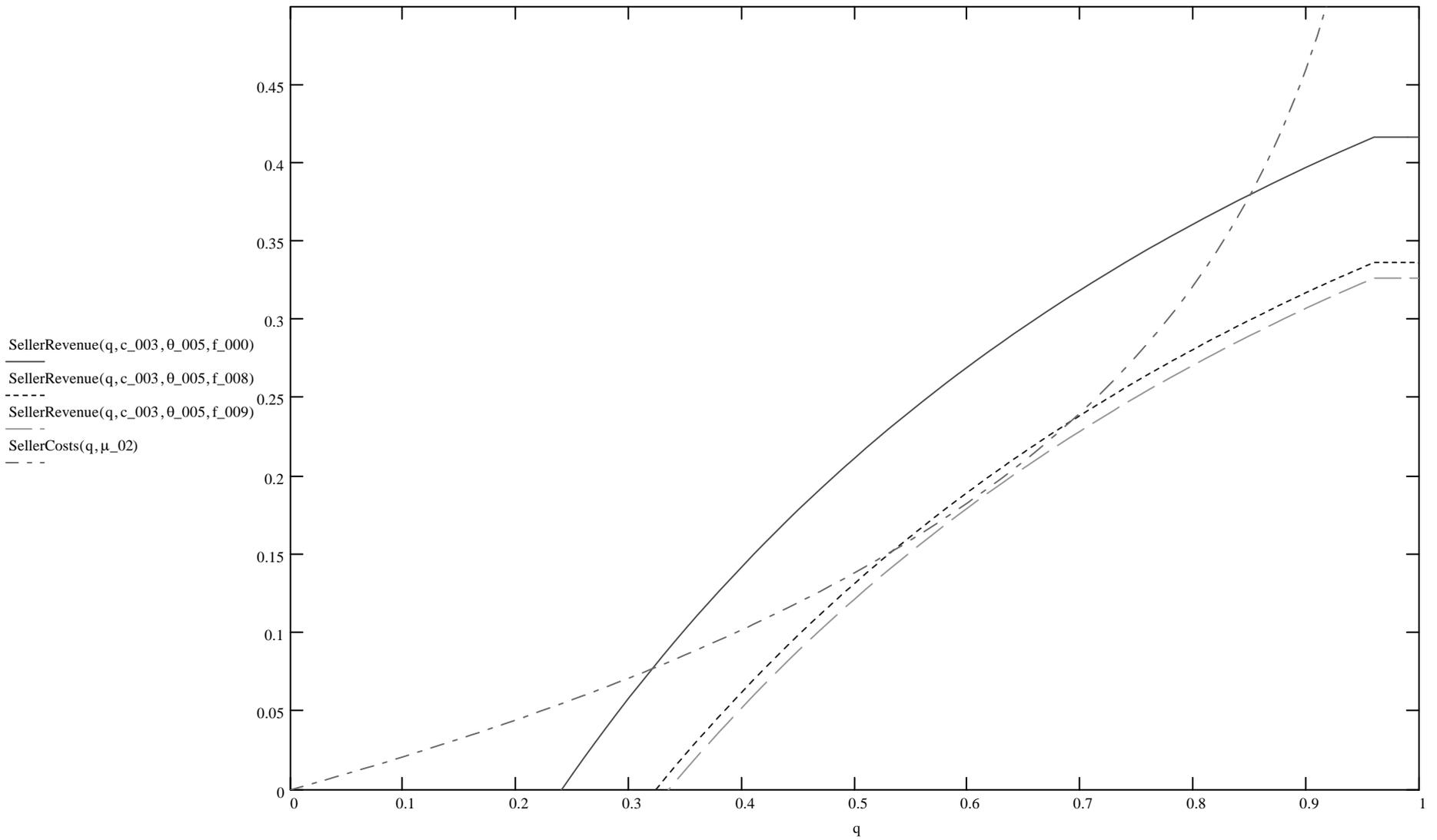


Figure 18. Market destroying effects of marginal changes in the listing fee with exponentially distributed costs.

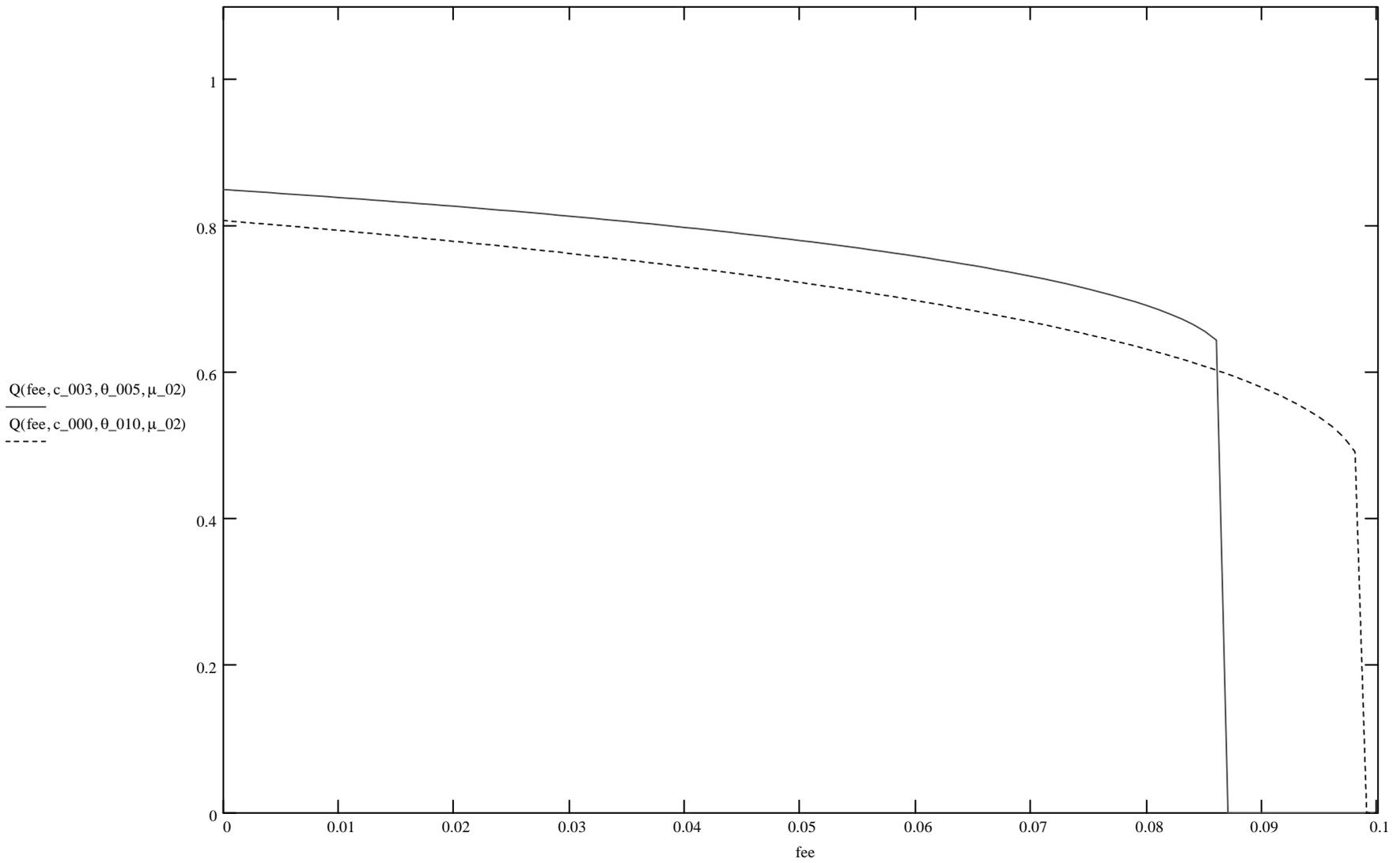


Figure 19. Auction hosting site demand curve with exponentially distributed seller costs.

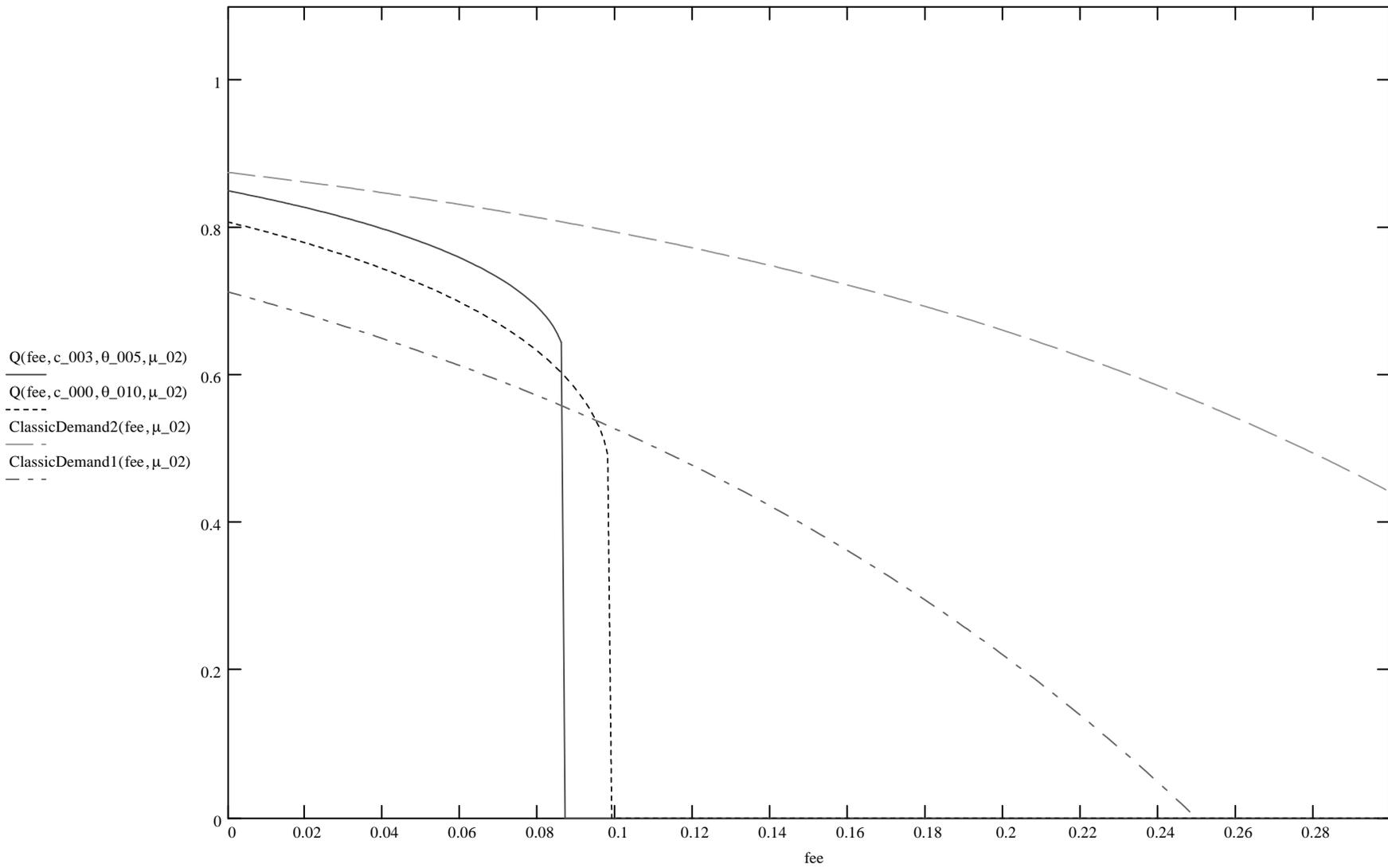


Figure 20. Comparison with "normal" demand curves with exponentially distributed seller costs.

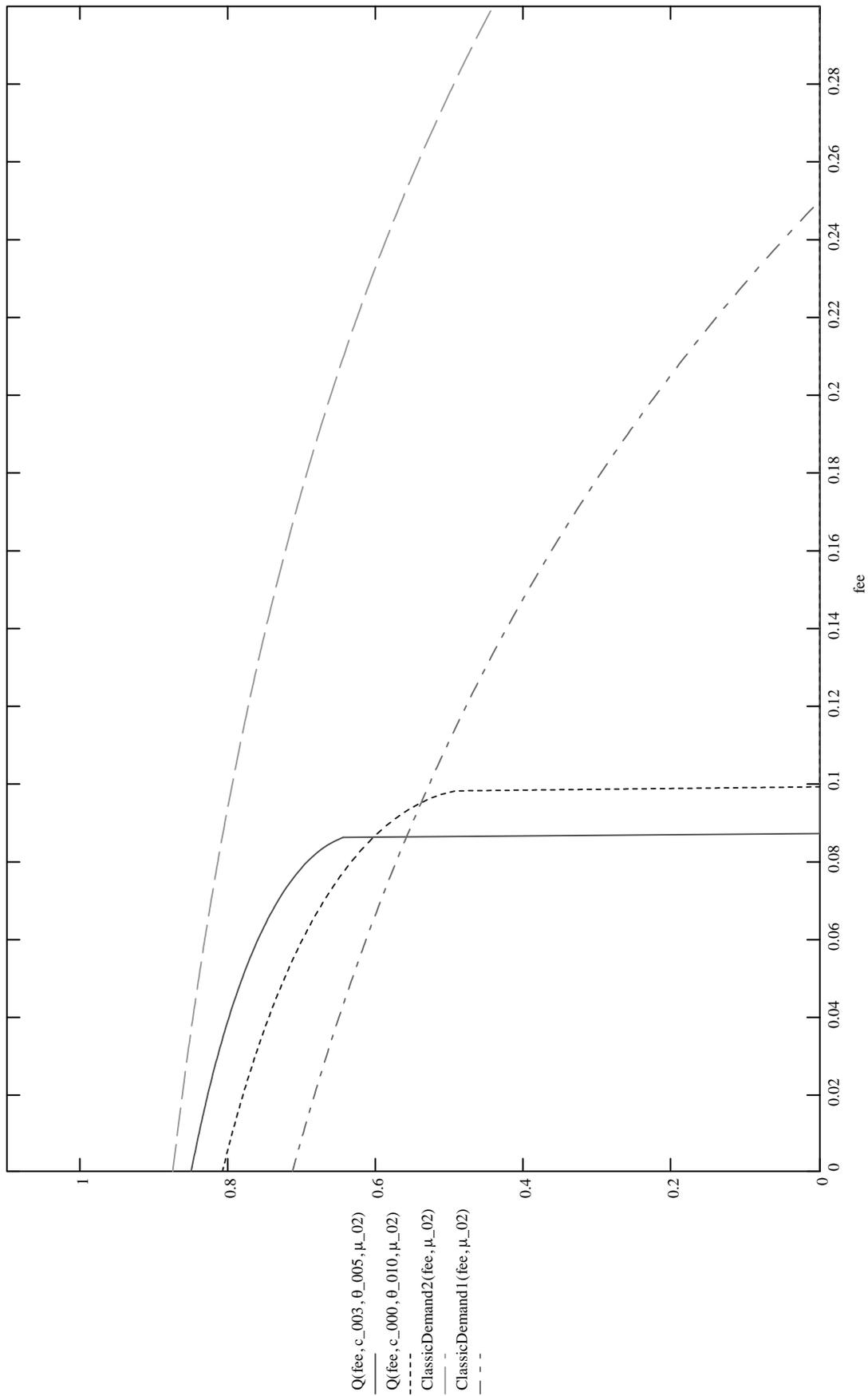


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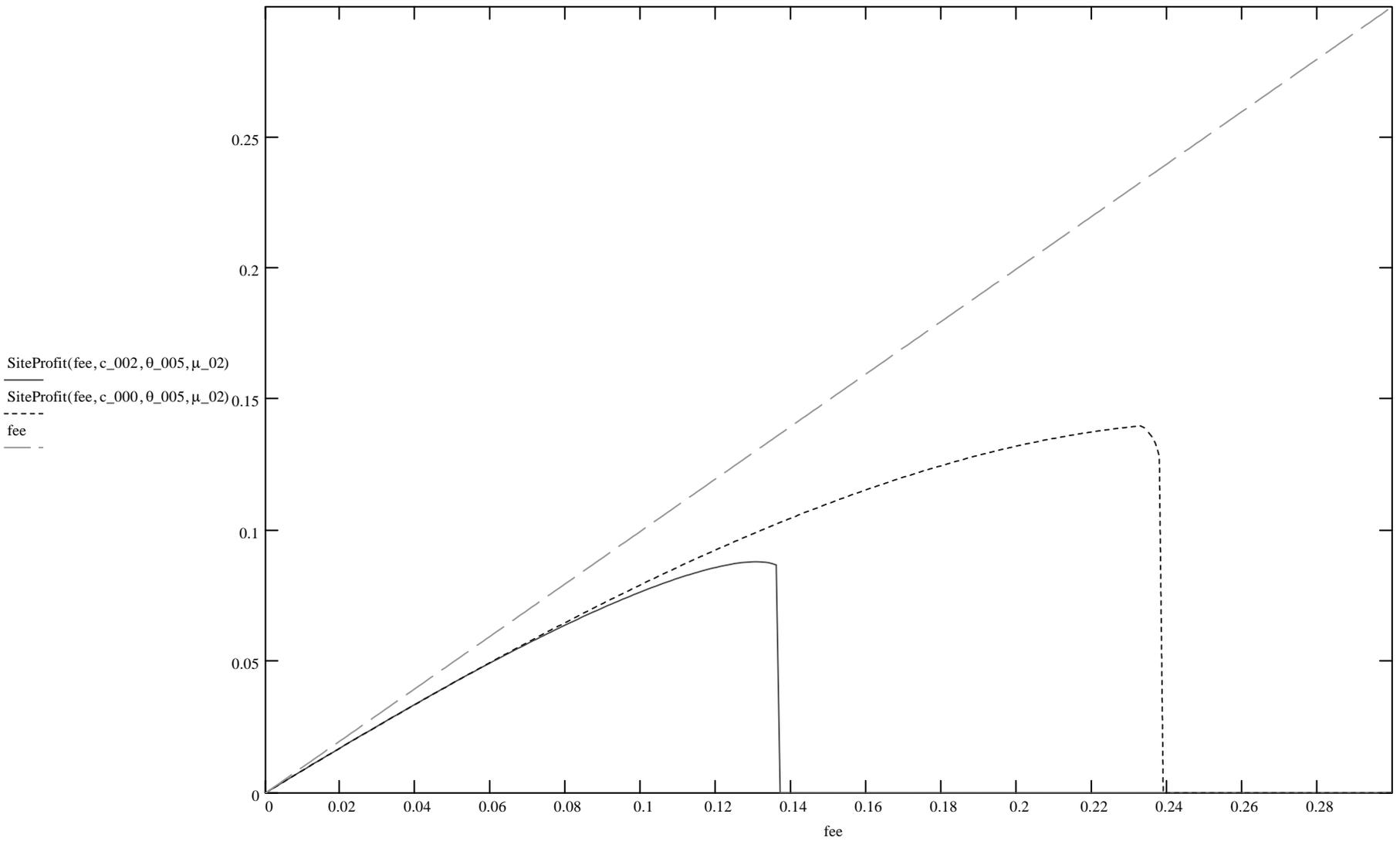


Figure 21. Site profit functions: changes in mean consumer attendance costs, exponentially distributed seller costs.

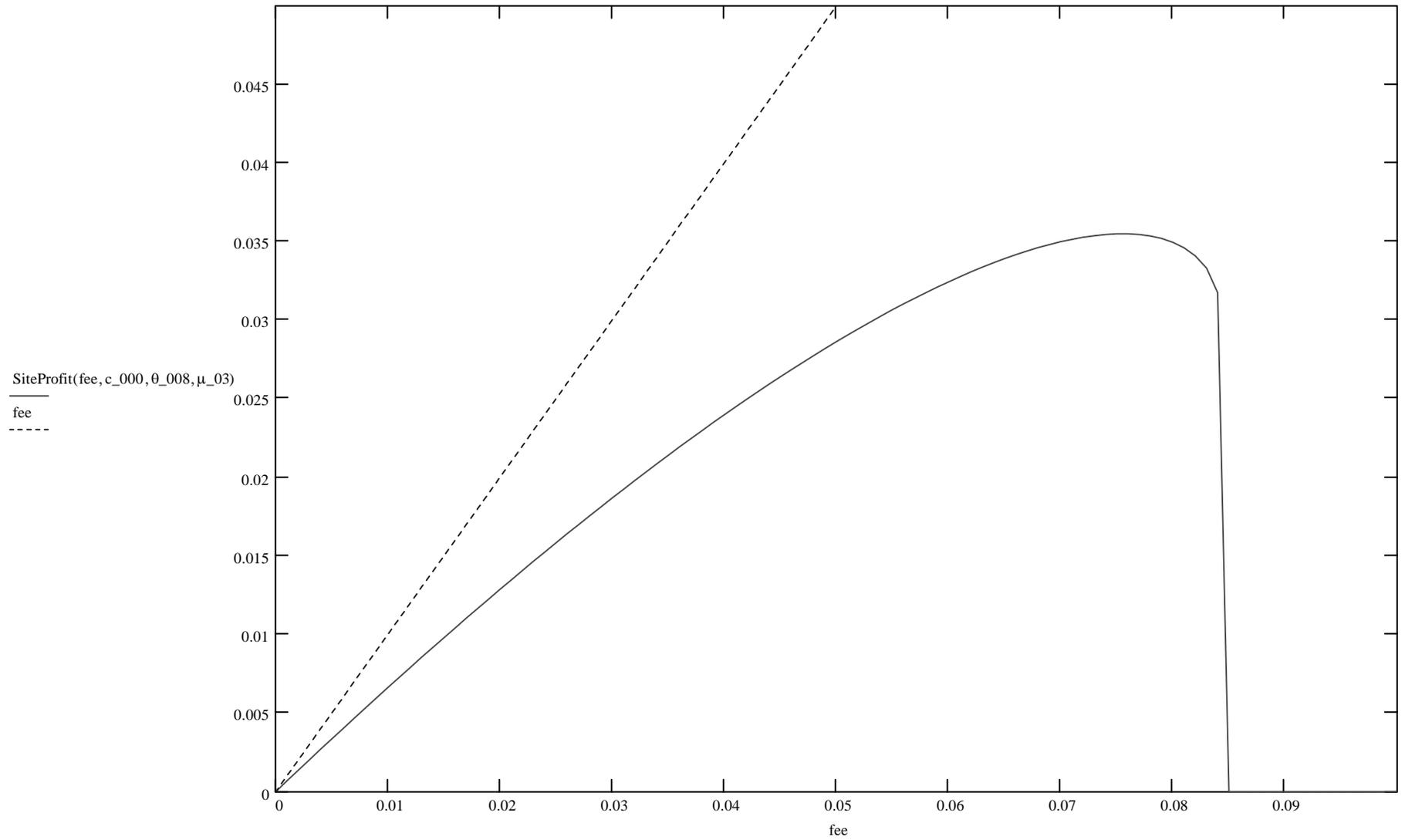


Figure 21b. A site profit function with an interior optimum ($q < 100\%$, differentiable maximum): Exponentially distributed costs case.

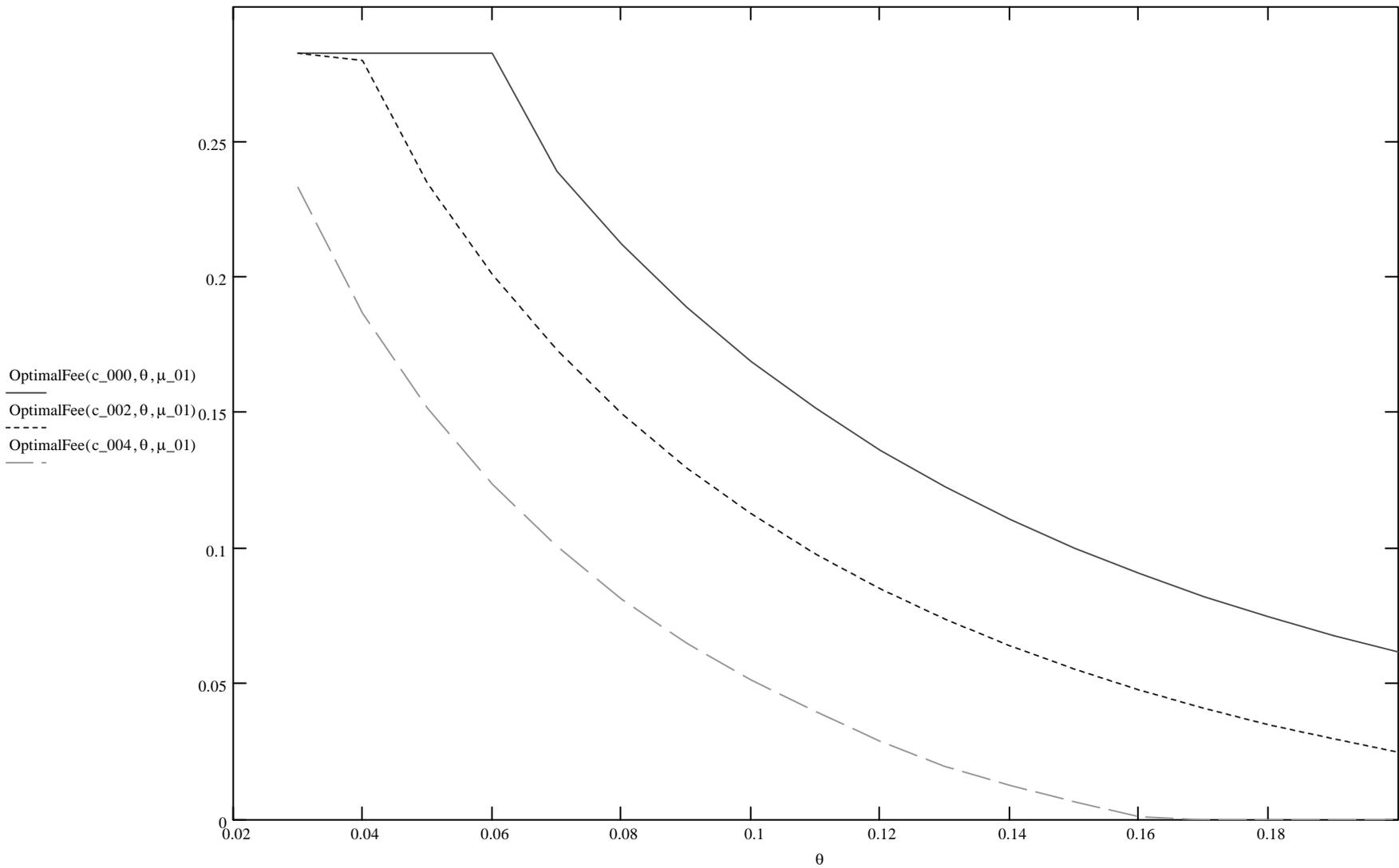


Figure 22. Optimal list fee as a function of the buyer participation costs: Exponentially distributed seller costs.

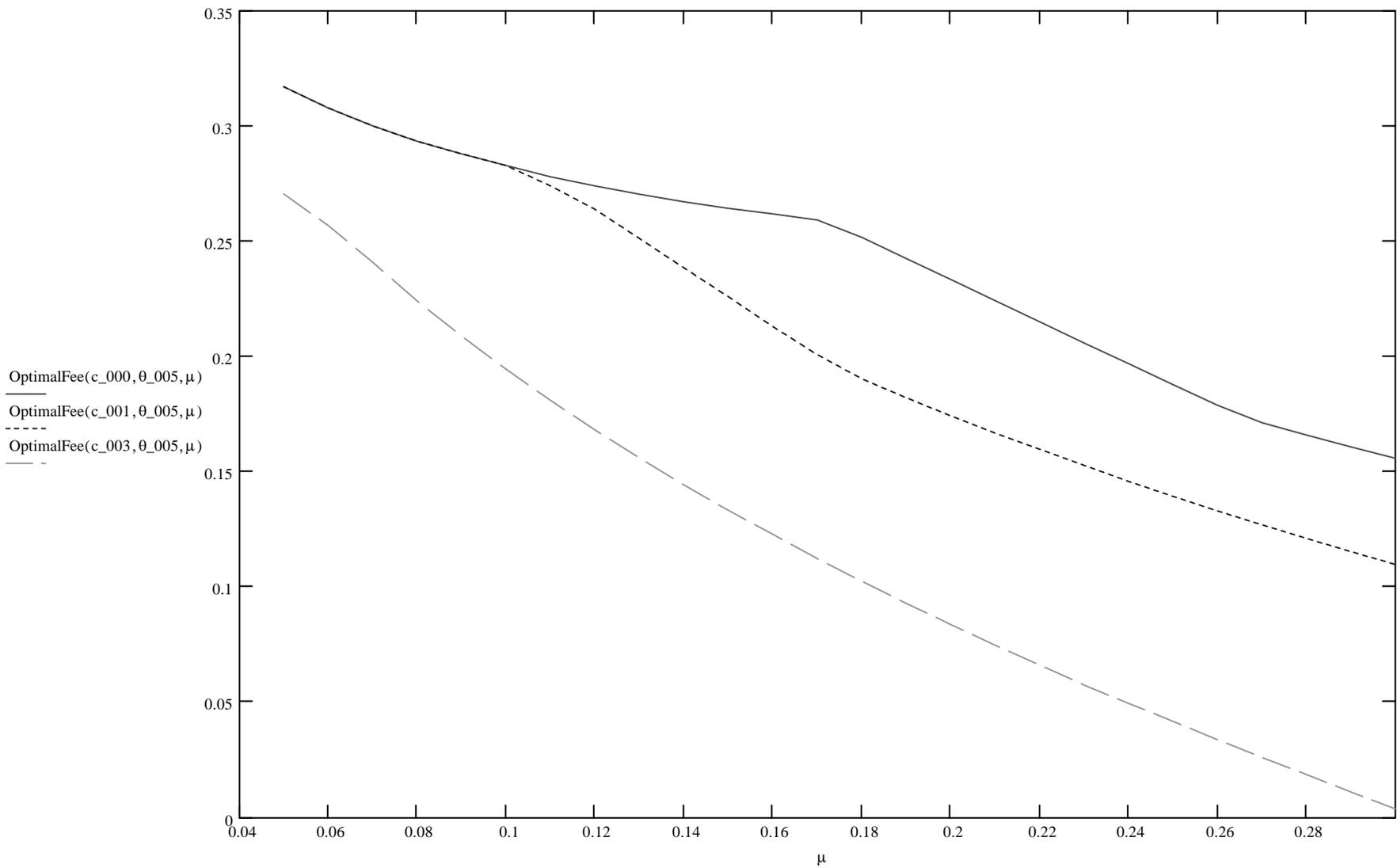


Figure 23. Optimal list fee as a function of the mean of seller participation costs: Exponentially distributed seller costs.

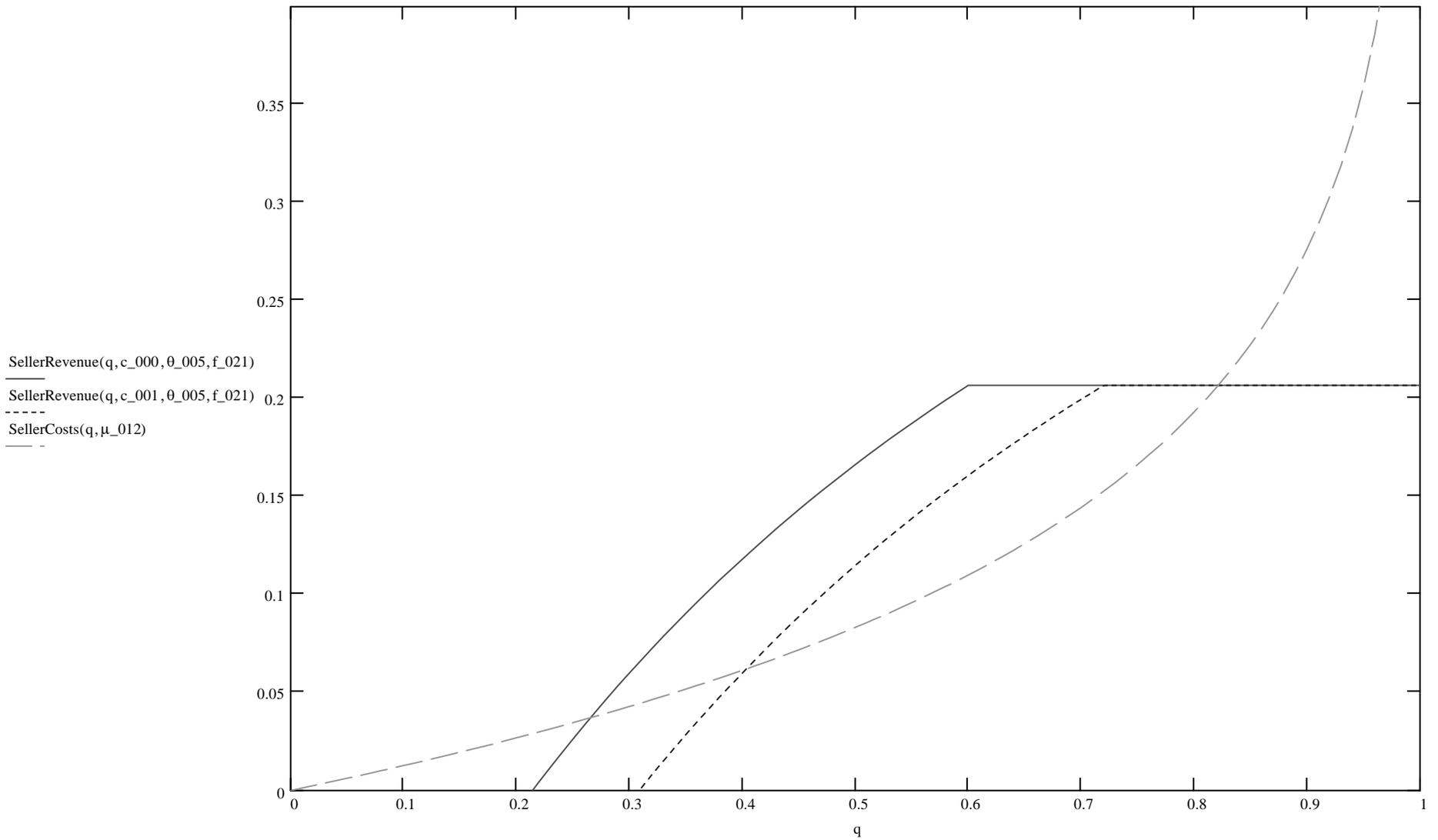


Figure 24. A closer look at the insensitivity of optimal fee to consumer attendance costs: Exponentially distributed seller costs.

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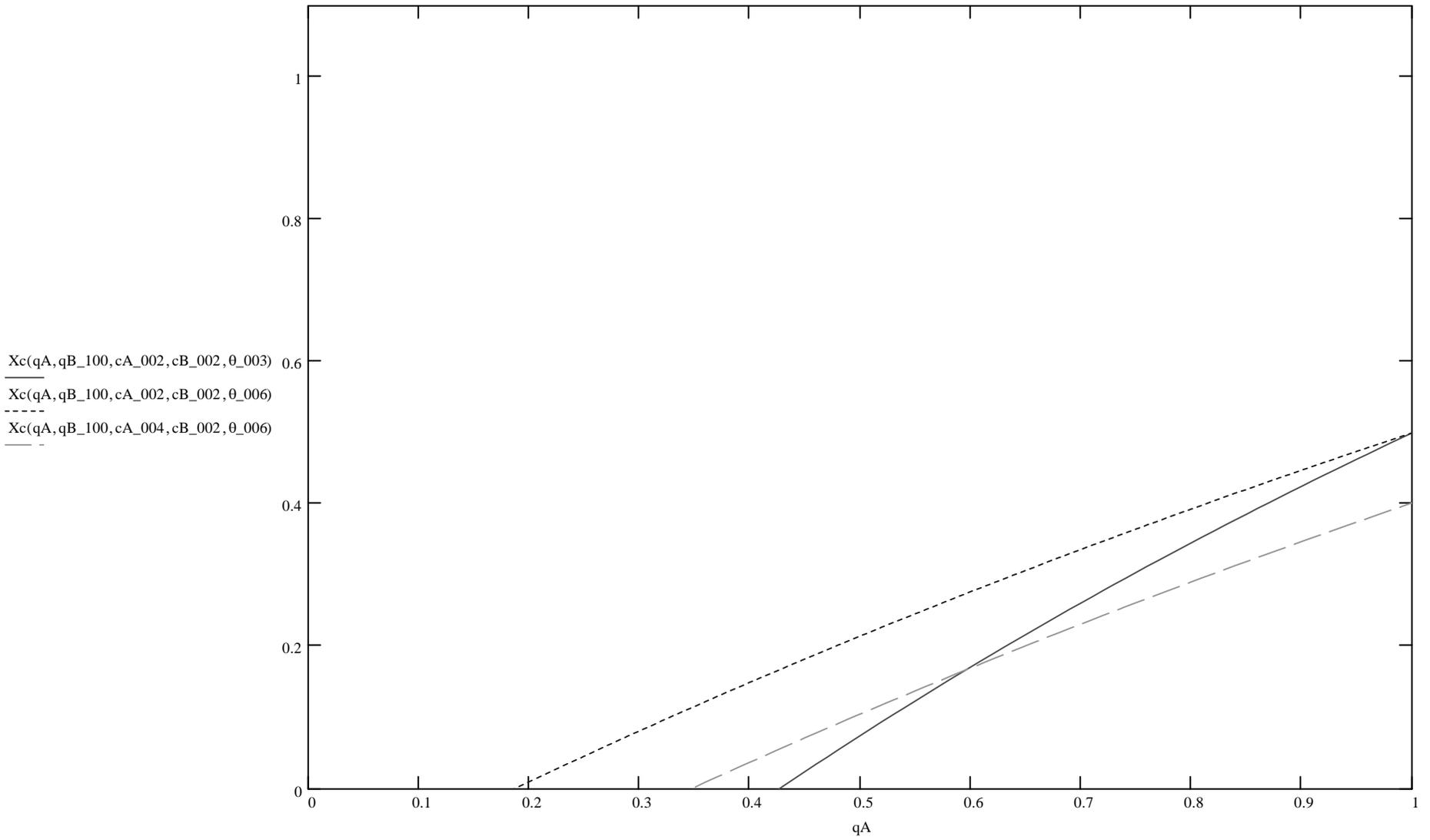


Figure 25. Location of critical consumer in duopoly.

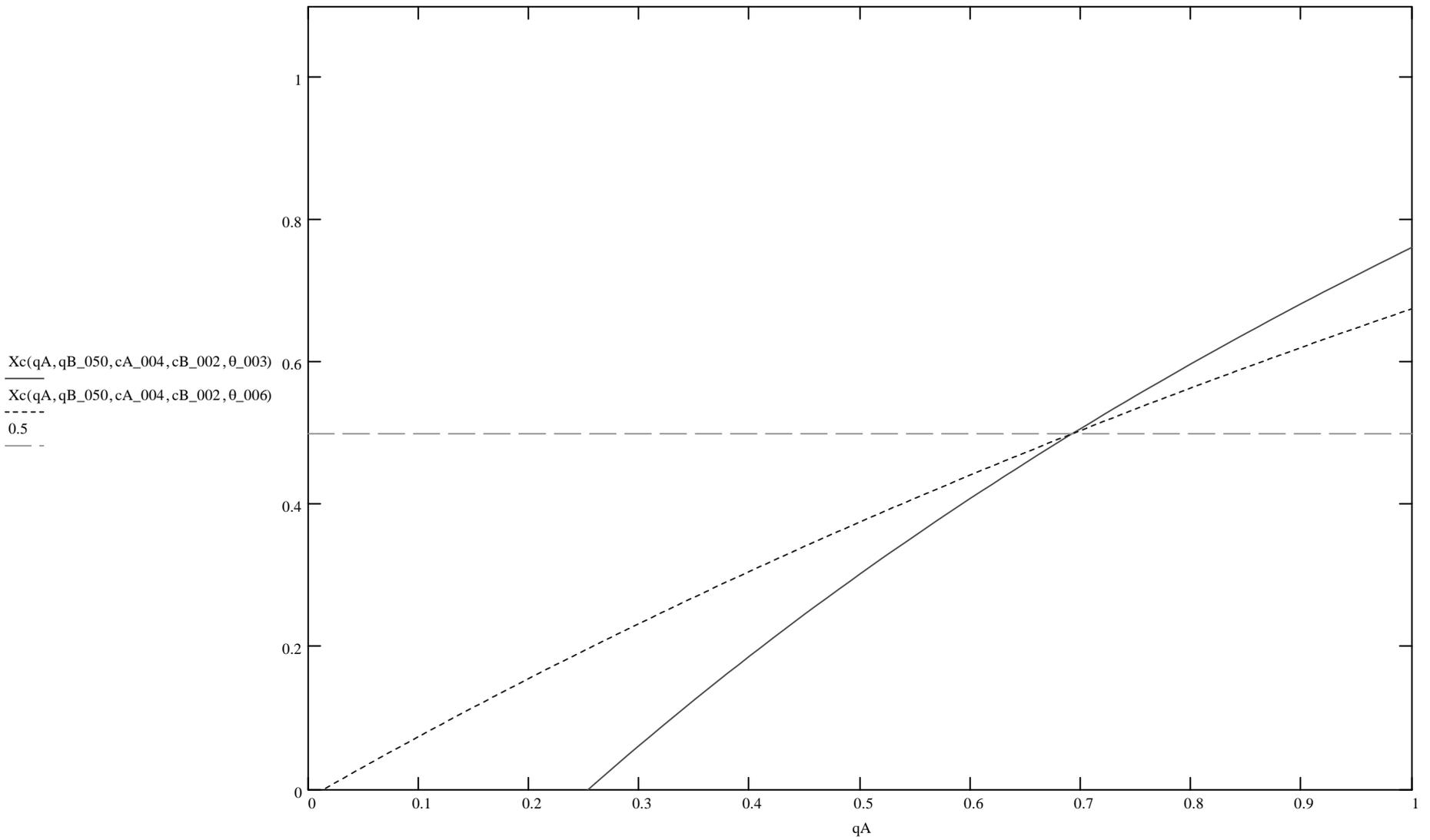


Figure 26. Location of critical consumer in duopoly: changes in theta.

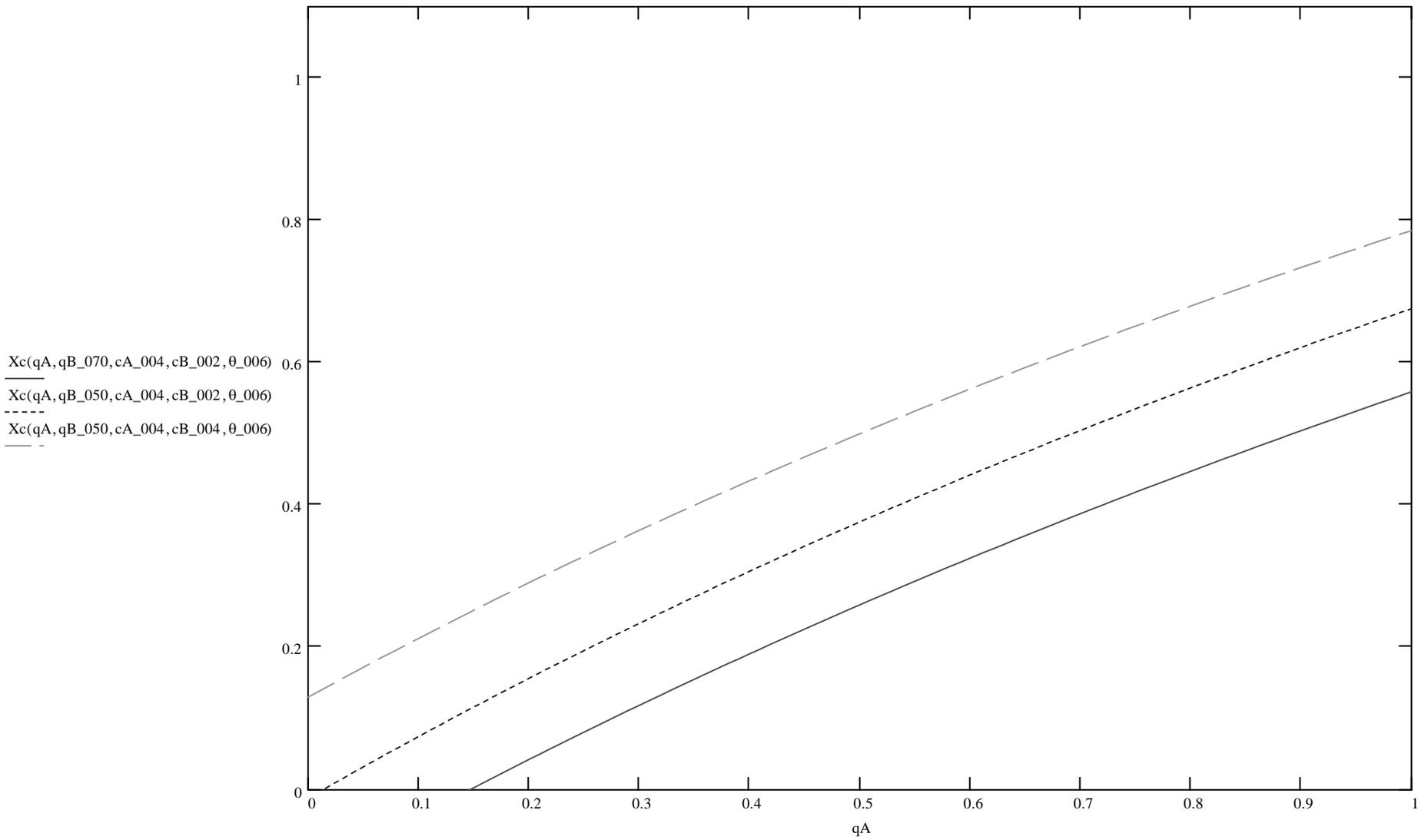


Figure 27. Location of critical consumer in duopoly: changes in other parameters.

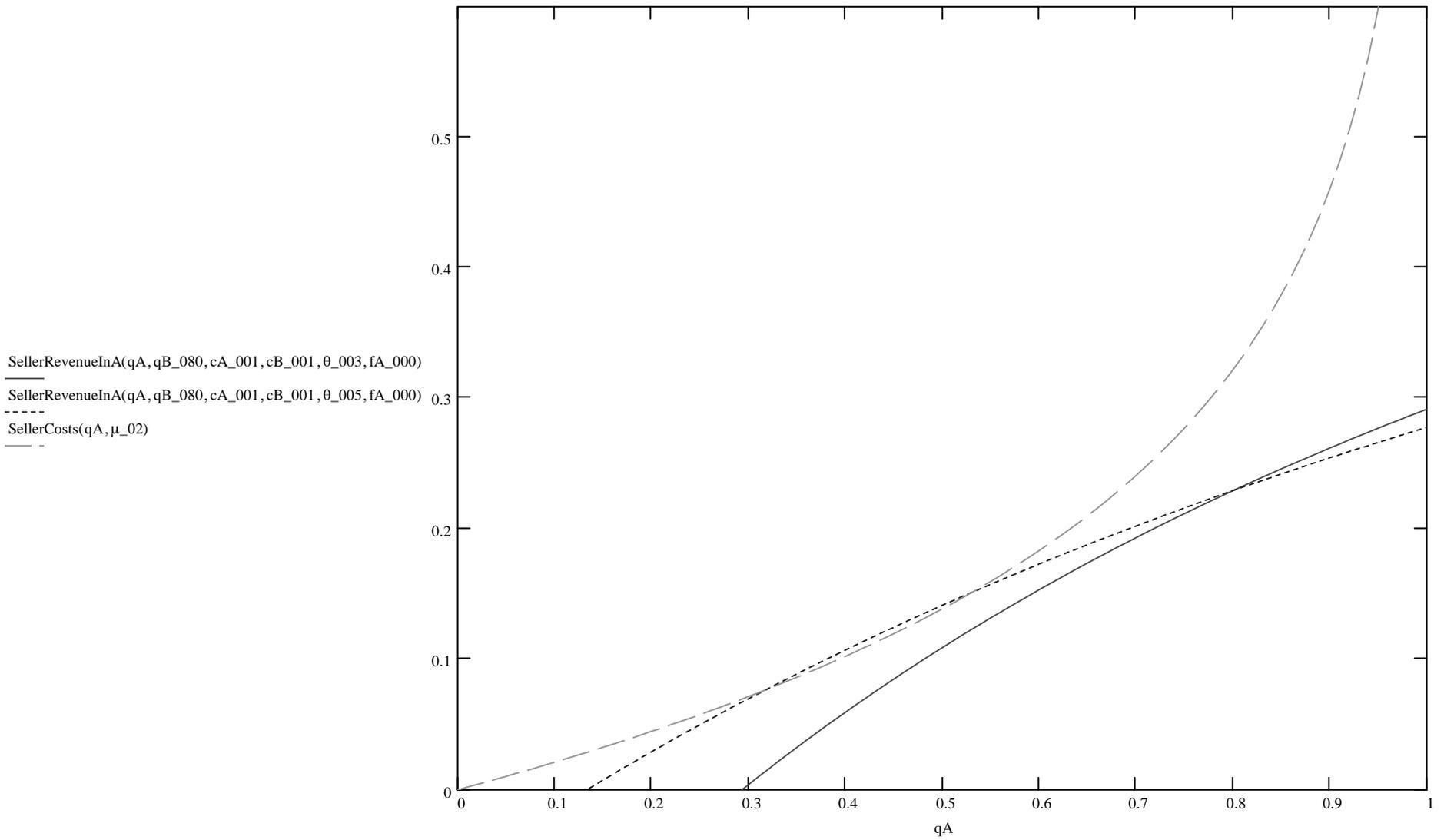


Figure 28. Site Equilibrium with exponentially distributed seller costs: Changes in theta.

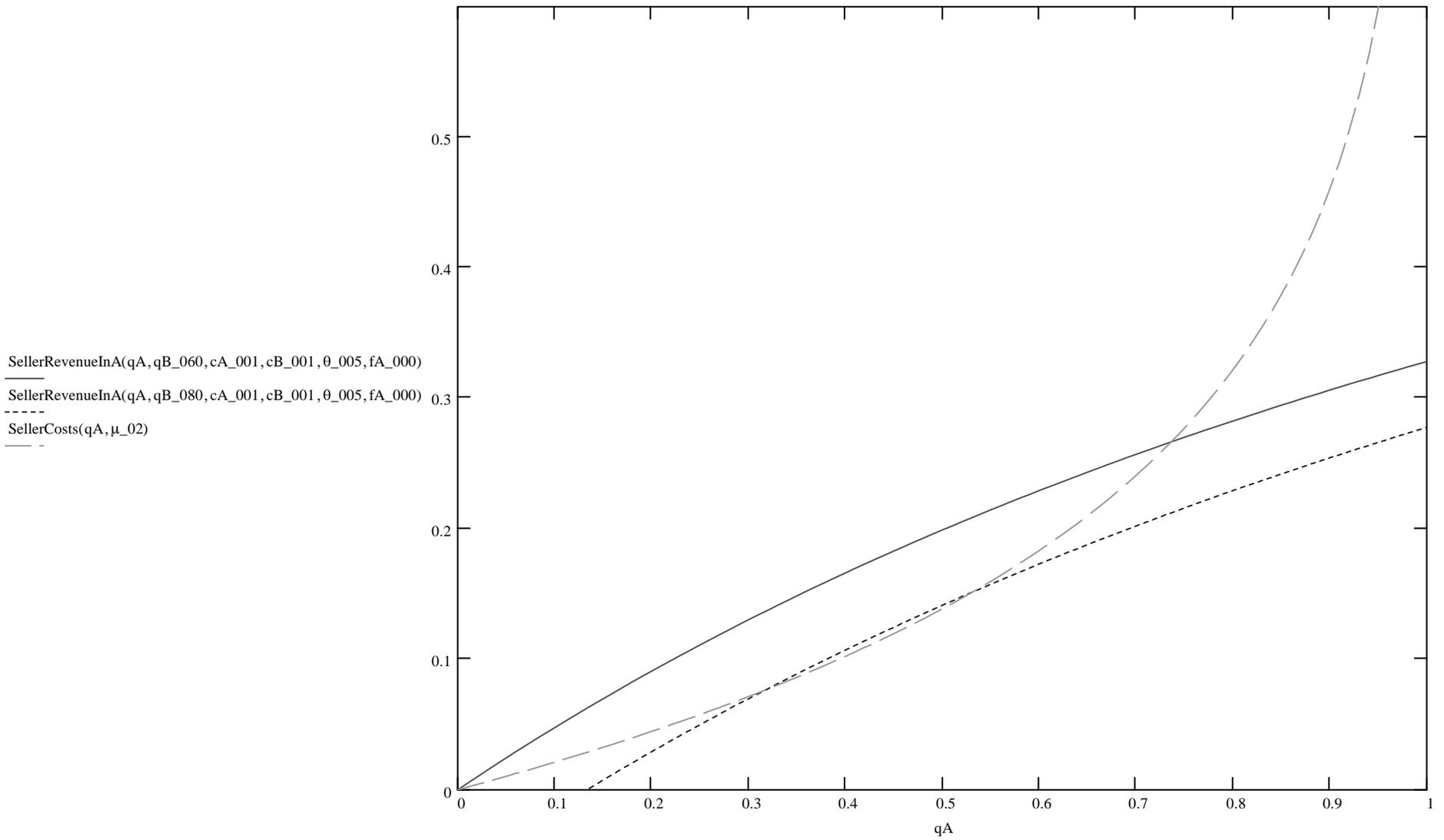


Figure 29. Site equilibrium with exponentially distributed seller costs: Changes in seller activity at competing site.

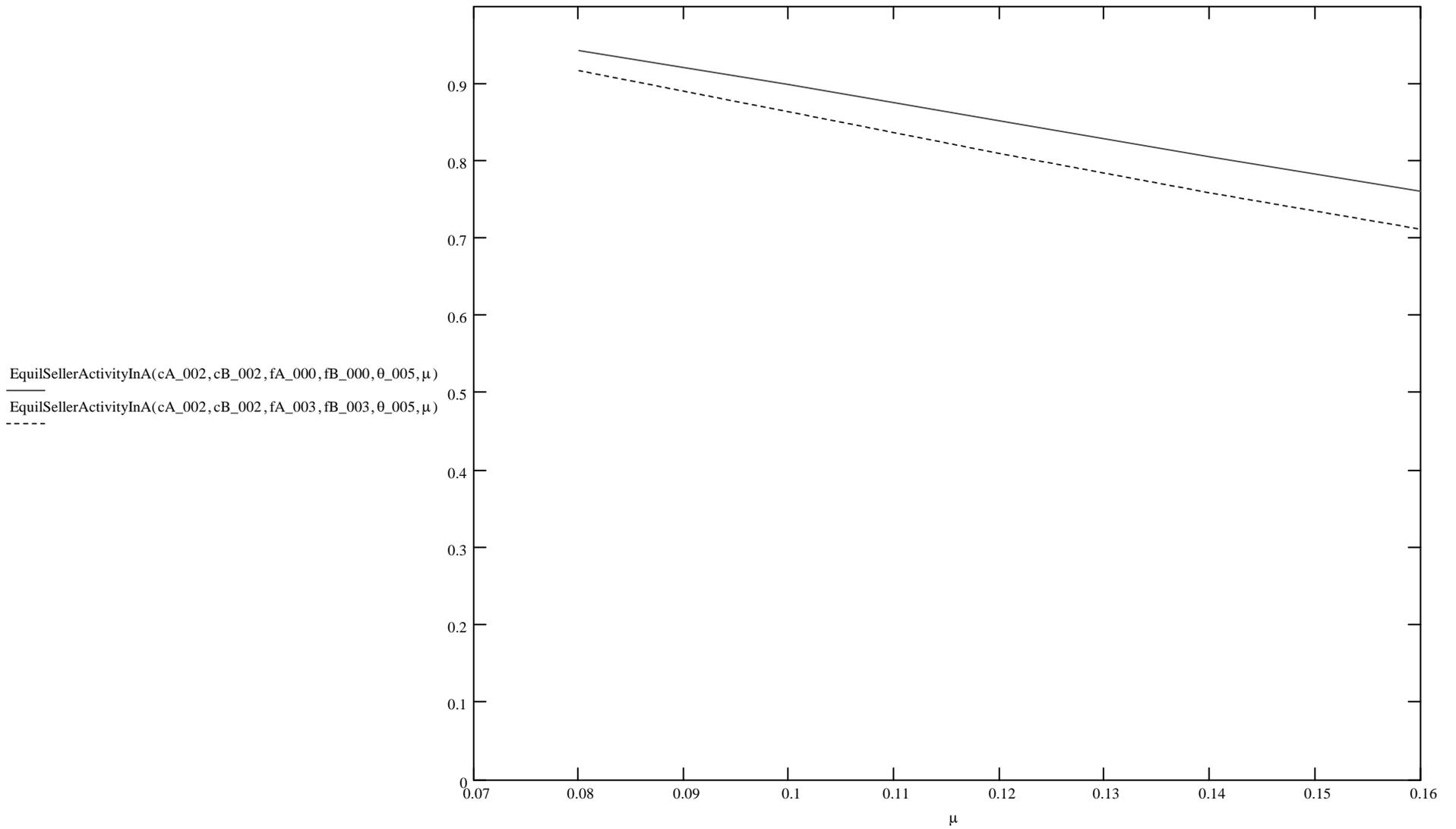


Figure 30. Equilibrium seller activity levels with exponentially distributed seller costs, as a function of mean seller costs: Symmetric sites.

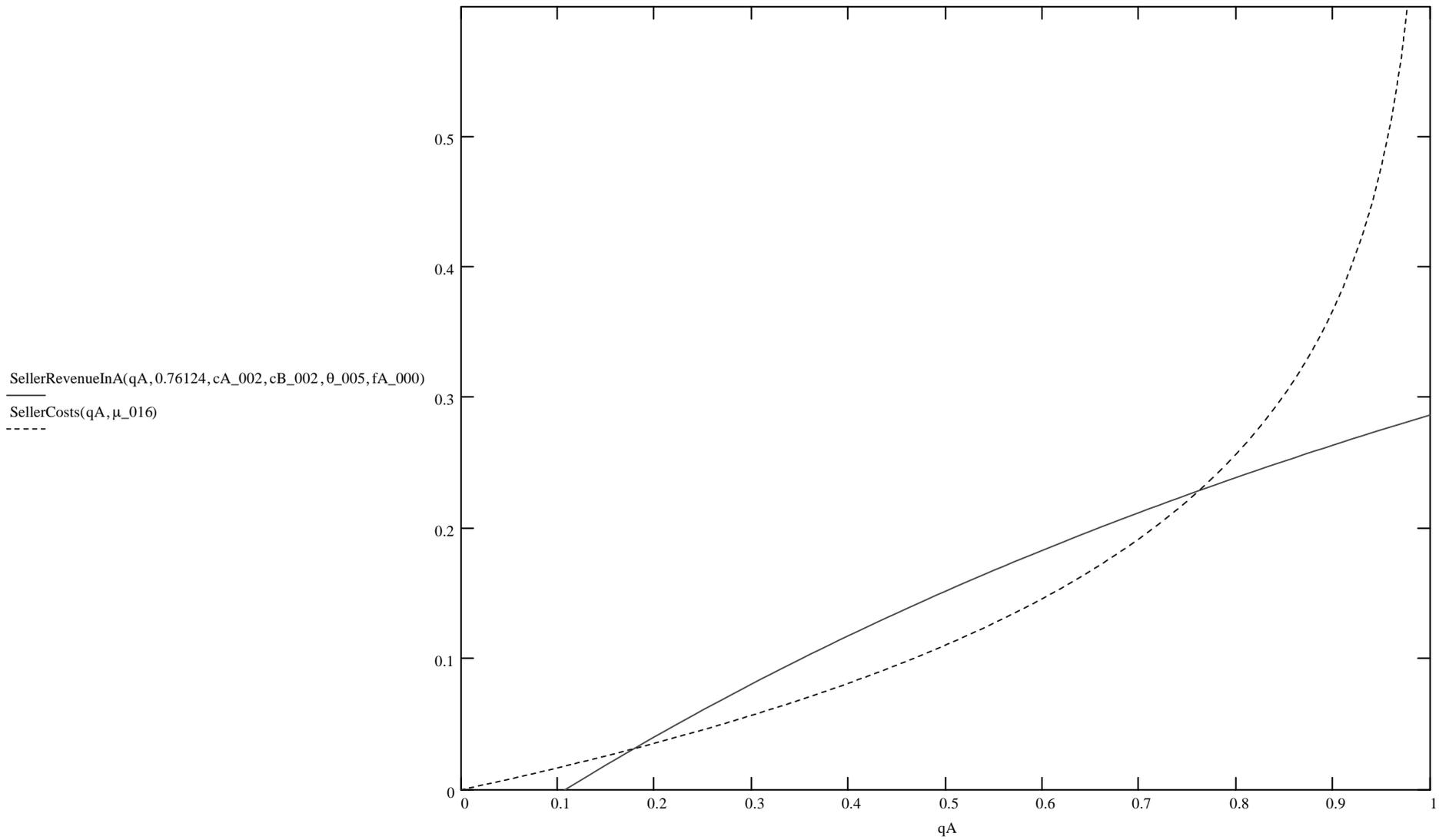


Figure 31. Symmetric equilibrium with exponentially distributed seller costs.

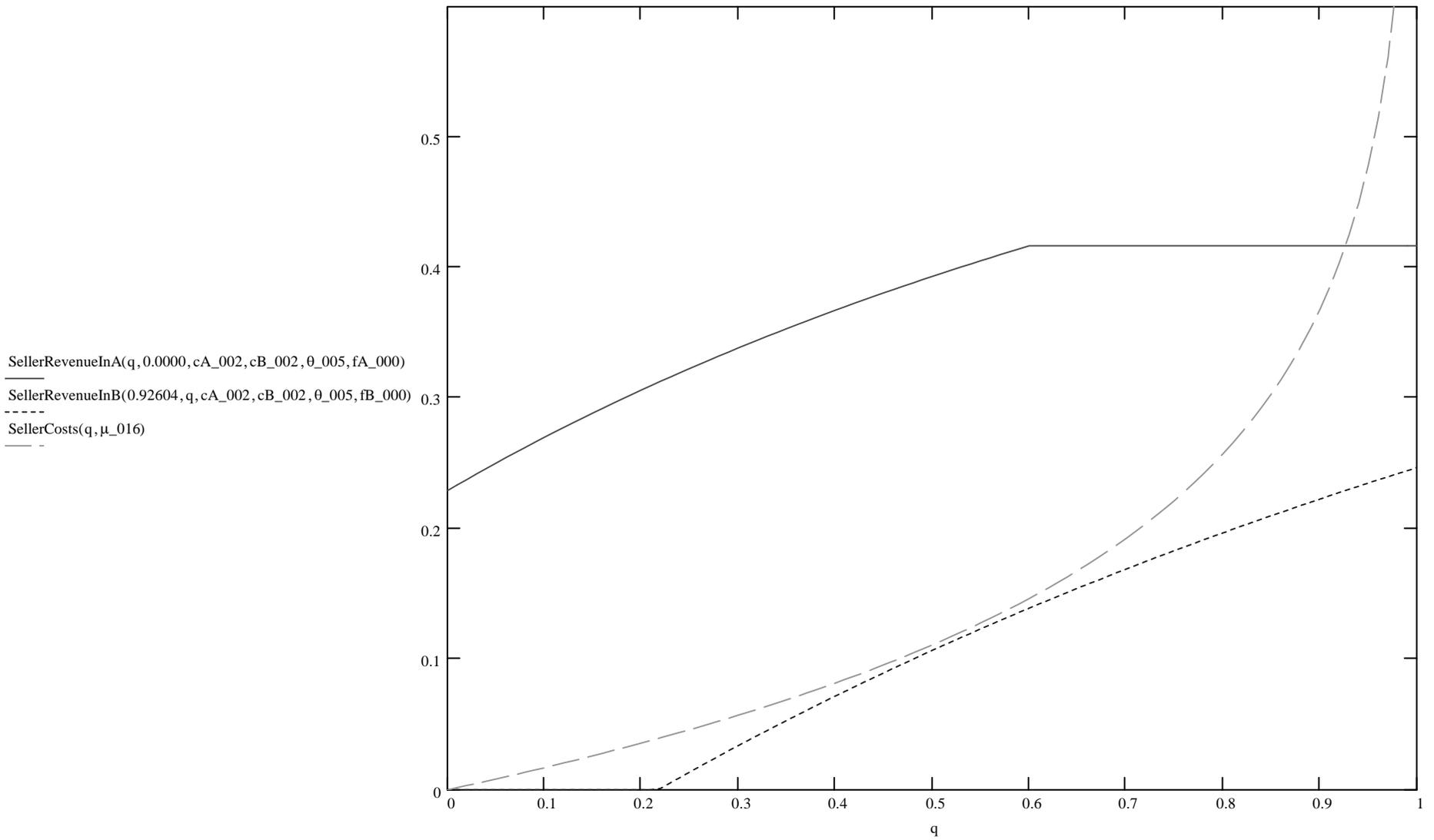


Figure 32. Asymmetric equilibrium with exponentially distributed seller costs.

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WORK IN PROGRESS/FUTURE WORK

1. Does reserve-setting endow the auction hosting site with additional pricing power?

That is, does a reserve change lead to a non-parallel shift in the revenue function?

2. Welfare implications of monopoly pricing and merger to monopoly.

Are welfare losses big? Are they even negative?

Not clear because value of the auction hosting site as a product increases in seller (and buyer) concentration.

3. Site competition for sellers.

How does lack of monopoly power on the seller side of the market affect conclusions (if at all)?