

Forward-looking bidding in online auctions

Robert Zeithammer
Graduate School of Business
University of Chicago



Let's buy a digital camera on eBay...



Canon S30, 15 mins left

Canon S40, 33 mins left

Olympus D40, 45 mins left

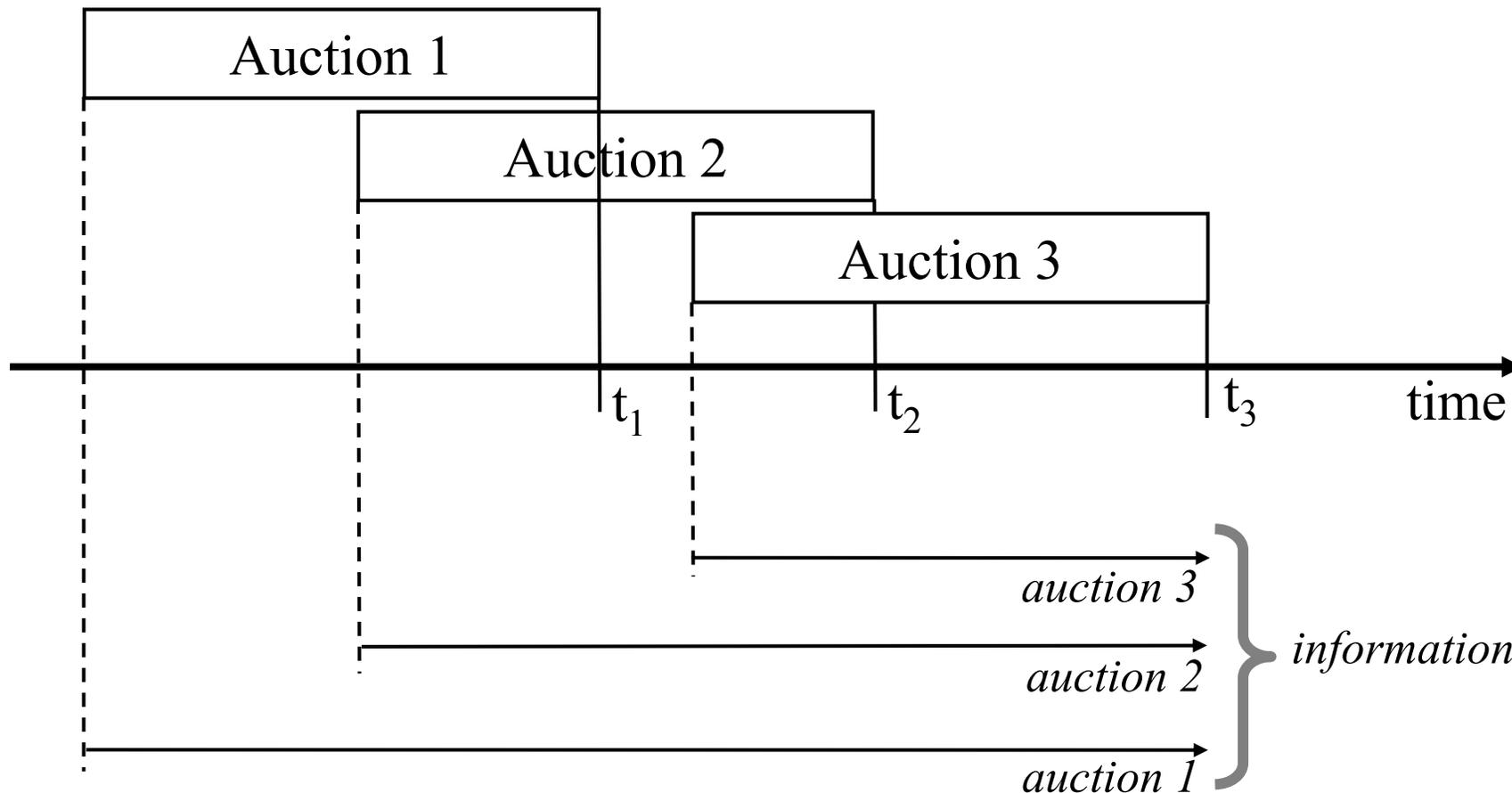
Canon S30, 47 mins left

Olympus D40, 53 mins left



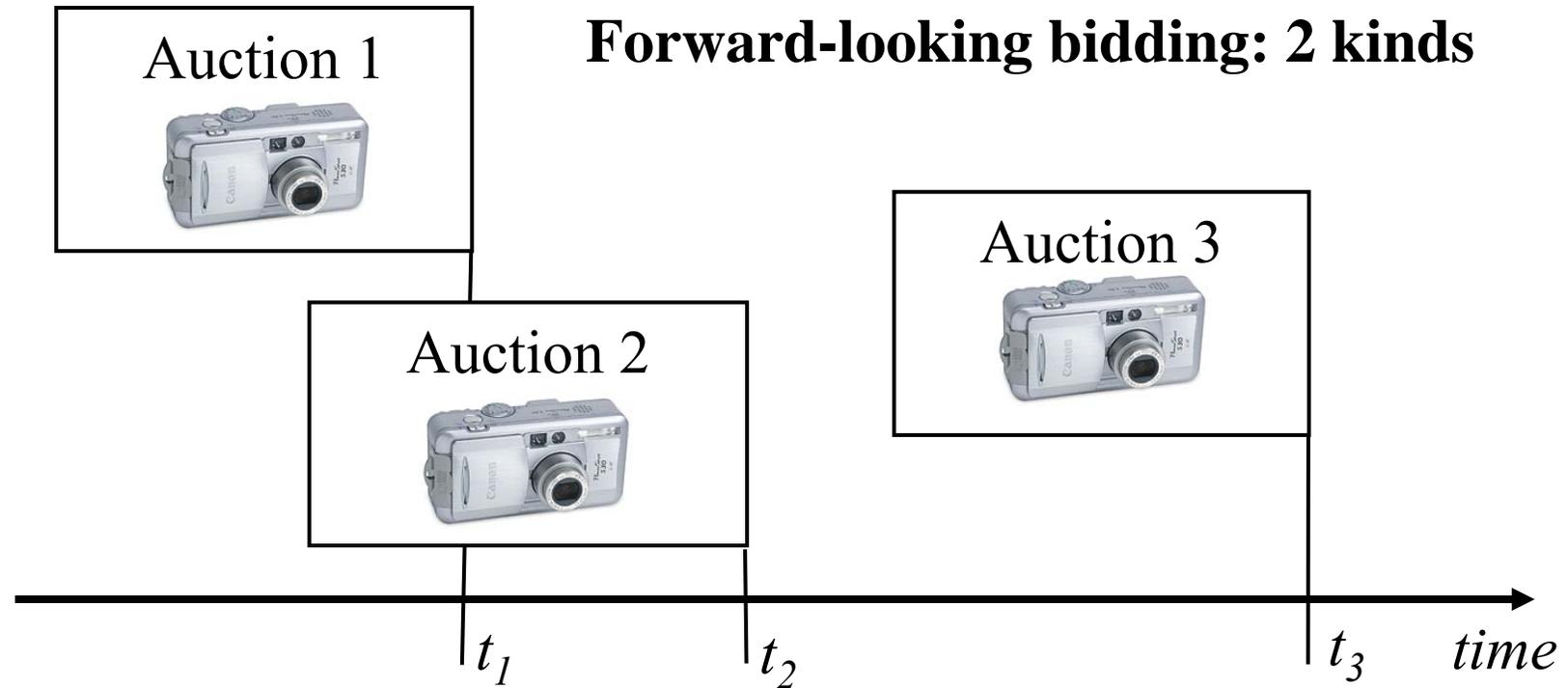
- Electronics, movies, computers ... each buyer only wants one unit
- Population heterogeneity in preferences (I am shopping for Canon S30)
- Simultaneous? No, sequential, implicitly organized by end time
- Interlaced sequences of auctions for essentially identical objects

eBay: sequential auctions with overlapping information



Research questions:

- 1) How to bid while incorporating the available information?
- 2) Do eBay bidders bid consistently with the theory?



unit-demand → **option-value of losing** → bid-shading (below isolated auction)

How to bid in auction 1?

- given the known (“**forward-seen**”) auction 2
- given a potential (“yet unseen”) auction 3 (Jofre-Bonet & Pesendorfer 03)

Some related work (all unit-demand bidders)

- **Milgrom & Weber (82b,99)** :
 - finite sequences , identical units
 - no use for information about future auctions (all the same)
 - finite \rightarrow no bidder-replacement needed \rightarrow elegant solution
- **Engelbrecht-Wiggans (94) , Jofre-Bonet & Pesendorfer (03)** :
 - finite sequences, stochastically equivalent units (different but *iid* units)
 - no information about future auctions \rightarrow symmetric and independent future
- **Gale & Hausch (94)** :
 - two auctions, different and potentially correlated units
 - $(v_1, v_2) \sim$ continuous F , both (v_1, v_2) known at the start
 - units not necessarily identical \rightarrow disposal issues
 - very hard to extend to many auctions
 - Contrast: I will only allow $v_i \in \{v, 0\} \approx \{"desired", "other"\}$

Model: One-period look-ahead, 2-type example

Infinite sequence of second-price, sealed-bid auctions

- varying waiting-times ω between individual auctions
- each auction sells one unit of a type- k good, $k: \{1,2\}$, $\Pr(k=1) = 1/2$
- no reserve

N_k bidders present in every period, live until win or exit ($\Pr(exit)=\lambda$ per hour)

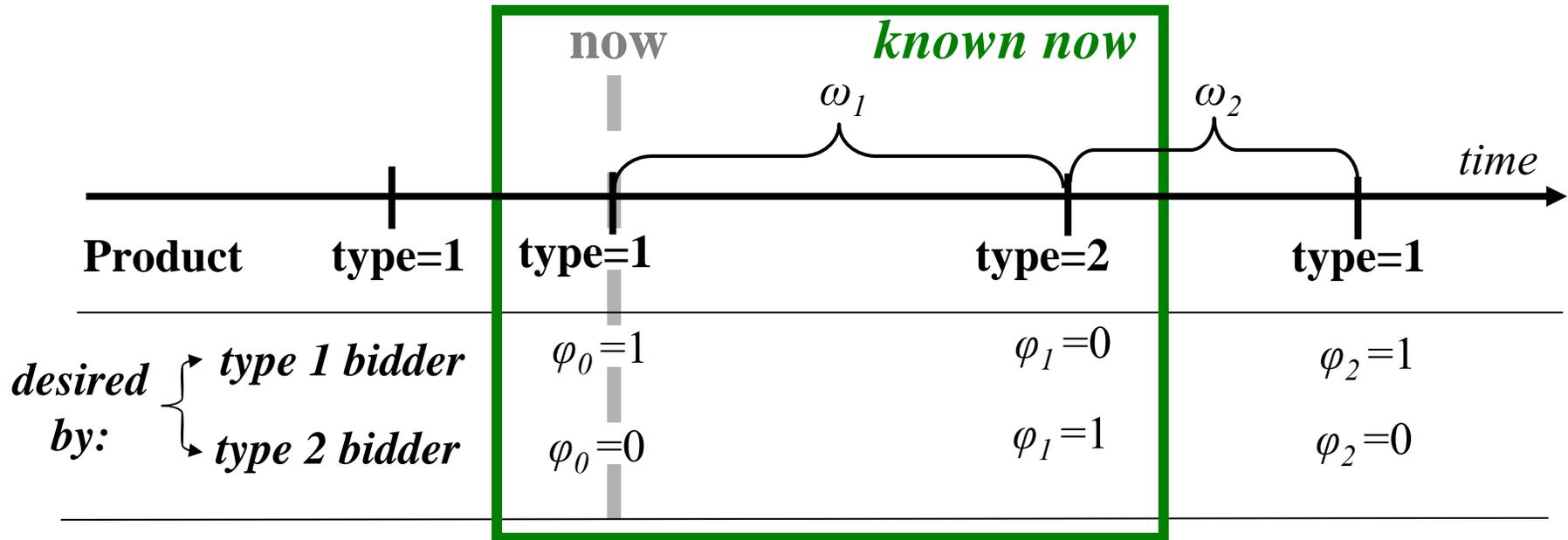
- unit-demand for only one type of good (“desired” type)
- IPV single-unit valuation of desired type, $v \sim F$ continuous
- **Info**: binary desirability of current unit φ_0 and next unit φ_1 , waiting-time ω_1

Everyone discounts future δ per hour, no memory

Discussion of the assumptions

- Interlaced sequences of identical-goods auctions with non-overlapping pop.
- Some bidder-replacement essential (otherwise steady-state survivors $v=0$)
- Innovation: bids depend on forward-seen information (ω_1, φ_1)

Model: One-period look-ahead, 2-type example



$S(\varphi_0, \varphi_1, \omega_1, v_i | c_0)$: expected surplus given loss to current competitive bid $c_0 \sim G$

$$b(\varphi_0, \varphi_1, \omega_1, v) = \arg \max_{\beta} \underbrace{\int_{\beta}^{\beta} (\varphi_0 v - c_0) dG(c_0)}_{\text{surplus if win now \& pay } c_0} + (\delta \lambda)^{\omega_1} \underbrace{\int_{\beta}^{\beta} S(\varphi_0, \varphi_1, \omega_1, v | c_0) dG(c_0)}_{\text{surplus if lose now to a bid } c_0}$$

$\mathbf{1}$ (current desired) \swarrow
 $\mathbf{1}$ (next desired) \searrow
 time till next \swarrow
 valuation of desired \searrow

key tradeoff

Optimal Forward-Seeing Bidding

$$b(\varphi_0, \varphi_1, \omega_1, v) = \arg \max_{\beta \geq 0} \int_0^{\beta} (\varphi_0 v - c_0) dG(c_0) + (\delta \lambda)^{\omega_1} \int_{\beta} S(\varphi_0, \varphi_1, \omega_1, v | c_0) dG(c_0)$$

$$\text{FOC: } b(1, \varphi_1, \omega_1, v) = v - (\delta \lambda)^{\omega_1} S(1, \varphi_1, \omega_1, v | c_0 = b(1, \varphi_1, \omega_1, v)) < v$$
$$b(0, \varphi_1, \omega_1, v) = 0$$

$$\text{SOC: } \frac{\partial S(1, \varphi_1, \omega_1, v | c_0)}{\partial c_0} > -\frac{1}{(\lambda \delta)^{\omega_1}}$$

Properties:

- can show FOC has a unique solution, and that SOC satisfied
- bid-shading (a benefit to losing compared to isolated 2PSB)
- “pivotal thinking” : bid as if about to lose in a tie to a bidder like you

Equilibrium

Bellman condition: In a symmetric pure-strategy Markov-Perfect equilibrium, the expected surplus function must be “correct”:

$$S(\varphi_{0,1}, \omega_1, v | c_0) = E_{\varphi_2, \omega_2} \left[\int_{b(\varphi_{1,2}, \omega_2, v)}^{b(\varphi_{1,2}, \omega_2, v)} (v - c_1) dG(c_1 | c_0, \varphi_{0,1,2}, \omega_{1,2}) + (\delta\lambda)^{\omega_2} \int_{b(\varphi_{1,2}, \omega_2, v)} S(\varphi_{1,2}, \omega_2, v | c_1) dG(c_1 | c_0, \varphi_{0,1,2}, \omega_{1,2}) \right]$$

S exists when F has a continuous density on a compact interval.

For a given F , S can be obtained by value-function iteration.

Could this be a basis for a structural approach?

Bidders are not price-takers, take into account evolution of the pool of competitors.

Properties of equilibrium bidding

$$b(\varphi_0, \varphi_1, \omega_1, \nu)$$

Empirical strategy:

- positive only on desired type: $b = 0 \leftrightarrow \varphi_0 = 0$

} *assume*
(identification)

- increase in waiting time ω_1

- decrease in desirability of the forward-seen type φ_1 (1 vs. 0)

} *test*

- increasing in ν on desired type

} *look at order*
stats given N

Reduced-form test of model predictions

- 1) $K+1$ types, multi-period look-ahead with timing (**type-independent**) information Ω and product (**type-specific**) information Φ
 - eBay bidders usually see about a week ahead, could be many periods
 - Ω : auctions ending within the next hour marked in **red**, easy to see
- 2) Focus on a particular subset x of the state-variables (Ω, Φ) and integrate out the rest of the state, i.e. generate “on average” predictions given x :
$$\bar{b}(x, v) = E[b(1, \Phi, \Omega, v) | x]$$
 (example: $x = \#$ auctions ending within next hour)
- 3) If something is true for every valuation v , it will be true for the order-statistics of the valuations within each auction (keeping N constant)
- 4) Note that the first and second highest bids are observed in eBay data. => Regress bid order-statistics $b_{(j)}(x)$ on x (control for varying N)

Reduced-form test of forward-seeing bidding

Forward-seeing variables considered:

type-independent Ω :

- number of category auctions ending in the next hour

type-specific Φ :

- 1) time until next auction of the same type
- 2) 1(current type offered at least once within next five auctions)
- 3) {1(current type offered 1,2,3,4,5 auctions from now)}

} considered one
at a time

Regression specification:

$$\bar{b}_{(m),i} = \underbrace{\alpha_{m,type(i)}}_{\substack{\text{type/order} \\ \text{fixed-effect}}} + \underbrace{\beta_m \Omega_i}_{\substack{\text{type-indep.} \\ \text{forward-seeing}}} + \underbrace{\gamma_m \Phi_{i,type(i)}}_{\substack{\text{type-specific} \\ \text{forward-seeing}}} + \underbrace{\theta_m z_i}_{\text{controls:}} + \varepsilon_{m,i}$$

i : observation (listing) auction i sells type

m : order of the order-statistic (either 1 or 2)

- number of unique bidders
- seller reputation
- new vs. used dummy
- listing features (photo...)

Two different datasets from eBay

2 datasets

- 1 month of top 30 movies on DVD in 2002 (**type** = title), 3113 listings
- 4 months of MP3 players in 2001 (**type** = brand X model)
further split because prices vary a lot:
 - 15 Low-priced players (~\$70, +/- \$20), 1693 listings
 - 15 High-priced players (~\$180, +/- \$60), 2451 listings

Weaknesses of the data

- only seller-provided descriptions to identify types
- number of unique bidders not perfectly observed

→ 3 (datasets) x 2 (order-stats) x 3 (type-spec variables) = 18 regressions

Preliminary evidence for predicted behavior

- Most eventual winners won only one unit within the data-period (93% in MP3-players and 87% in movies).
- A substantial number of bidders participated in more than one auction (43% in MP3-players and 33% in movies) and those who did mostly stuck to bidding on one product-type.
- It does not seem that the multi-auction bidders simply submitted a very low bid initially to learn about the auction process or their true valuation, and only later raised their bid to their “full” willingness to pay. (Of the multi-bidders, 49% in movies and 59% in MP3 players submitted a higher second bid).

Regression results

Predicted effects : Number within category in next hour ↓, Time until next identical ↑, Identical in next 5 auctions ↓, More distant future options gradually less effect.

DVD movies

type-independent: mostly not significant, predicted sign

type-specific : all as predicted:

- Average price ~\$10 → effect size on price: 3-7%

MP 3 players

type-independent: as predicted, but small (double number of auctions in next hour ~ 2 % ↓)

type-specific :

- Low-price players: not significant, predicted sign
- High-priced players : all as predicted
- Average price ~\$180 → effect size on price: 4-6% when the same type is available in the next 5 auctions, 1% when next delayed by 1 hour.

Regularity: 2nd highest bid (price) exhibits bigger effects than 1st highest bid. (?)

Results: movies

Variable	Highest bid		2nd highest bid		Highest bid		2nd highest bid		Highest bid		2nd highest bid	
	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)
α (30 type-specific dummies)	suppressed for parsimony (mean 7.92, standard deviation 1.39, minimum 5.40, maximum 10.9)											
θ (top-seller dummy)	0.645	(9.30)	0.575	(8.51)	0.637	(9.33)	0.561	(8.41)	0.628	(9.18)	0.553	(8.31)
θ (new dummy)	0.756	(10.63)	0.826	(11.87)	0.747	(10.70)	0.817	(11.93)	0.748	(10.71)	0.825	(12.05)
θ (current competition)	0.128	(8.81)	0.094	(6.20)	0.127	(8.89)	0.096	(6.38)	0.125	(8.71)	0.094	(6.26)
β (log (# next hour+1))	-0.045	-(0.83)	0.033	(0.63)	-0.087	-(1.74)	-0.039	-(0.80)	-0.084	-(1.68)	-0.039	-(0.81)
γ (log time until next)	0.06	(2.35)	0.108	(4.07)								
γ (same type next 5 auctions)					-0.17	-(1.92)	-0.313	-(3.51)				
γ (same type 1 a. from now)									-0.348	-(2.52)	-0.722	-(5.00)
γ (same type 2 a. from now)									-0.433	-(2.10)	-0.385	-(1.93)
γ (same type 3 a. from now)									0.136	(0.67)	0.098	(0.48)
γ (same type 4 a. from now)									-0.051	-(0.29)	-0.182	-(1.05)
γ (same type 5 a. from now)									0.07	(0.31)	0.025	(0.12)
	N=3017	$R^2=0.42$	N=2356	$R^2=0.53$	N=3113	$R^2=0.42$	N=2431	$R^2=0.53$	N=3113	$R^2=0.42$	N=2431	$R^2=0.53$

- All three measures of **type-specific** future information as predicted
 - Time until next identical \uparrow , Identical in next 5 auctions \downarrow , More distant future options gradually less effect.
- **Type-independent** future information mostly not significant, predicted sign
- 2nd highest bid exhibits bigger effects than 1st highest bid. (?)
 - Average price \sim \$10, so effect size on price: 3-7%

Low-price players:	Highest bid		2nd highest bid		Highest bid		2nd highest bid		Highest bid		2nd highest bid	
Variable	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)
α (15 type-specific dummies)	suppressed for parsimony (mean 65-70, standard deviation 19-20, minimum 40, max 103-108)											
θ (log (Seller Reputation+6))	0.869	(2.92)	0.038	(0.16)	0.803	(2.79)	0.041	(0.17)	0.799	(2.77)	0.037	(0.16)
θ (photo-listing dummy)	1.855	(1.63)	0.207	(0.23)	2.04	(1.83)	0.428	(0.48)	2.065	(1.85)	0.483	(0.53)
θ (bold-listing dummy)	-1.318	-(0.53)	-1.673	-(0.88)	-0.996	-(0.41)	-1.664	-(0.89)	-0.996	-(0.40)	-1.605	-(0.86)
θ (gallery-listing dummy)	4.633	(3.03)	4.322	(3.56)	4.339	(2.90)	4.092	(3.43)	4.331	(2.89)	4.088	(3.42)
θ (new dummy)	2.951	(3.03)	4.264	(5.53)	3.215	(3.36)	4.378	(5.75)	3.217	(3.36)	4.369	(5.73)
θ (current competition)	0.28	(2.46)	0.537	(5.37)	0.287	(2.57)	0.544	(5.51)	0.286	(2.56)	0.544	(5.51)
β (log (# next hour+1))	-2.578	-(3.48)	-2.392	-(4.00)	-2.587	-(3.69)	-2.576	-(4.55)	-2.596	-(3.69)	-2.596	-(4.57)
γ (log time until next)	0.048	(0.15)	0.176	(0.70)								
γ (same type next 5 auctions)					-0.965	-(1.00)	-0.358	-(0.46)				
γ (same type 1 a. from now)									-0.454	-(0.34)	0.296	(0.27)
γ (same type 2 a. from now)									-1.376	-(0.68)	-0.359	-(0.23)
γ (same type 3 a. from now)									-1.436	-(0.73)	-0.221	-(0.14)
γ (same type 4 a. from now)									-1.586	-(0.73)	-1.914	-(1.07)
γ (same type 5 a. from now)									-0.88	-(0.39)	-1.046	-(0.56)
	N=1645	$R^2=0.63$	N=1600	$R^2=0.73$	N=1693	$R^2=0.63$	N=1646	$R^2=0.72$	N=1693	$R^2=0.63$	N=1646	$R^2=0.72$
High-price players:												
Variable	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)	Estimate	(t-stat)
α (15 type-specific dummies)	suppressed for parsimony (mean 186-171, standard deviation 57-58, minimum 99-114, max 316-334)											
θ (log (Seller Reputation+6))	0.729	(1.67)	1.332	(3.41)	0.816	(1.89)	1.405	(3.60)	0.787	(1.82)	1.392	(3.57)
θ (photo-listing dummy)	0.009	(0.01)	0.977	(0.85)	-0.162	-(0.13)	0.746	(0.65)	-0.102	-(0.08)	0.852	(0.74)
θ (bold-listing dummy)	4.427	(1.50)	2.597	(1.00)	4.369	(1.49)	2.898	(1.12)	4.176	(1.42)	2.693	(1.04)
θ (gallery-listing dummy)	1.088	(0.45)	-1.11	-(0.51)	0.966	(0.40)	-1.228	-(0.57)	1.146	(0.48)	-1.171	-(0.54)
θ (new dummy)	7.112	(5.07)	7.131	(5.65)	7.259	(5.21)	7.112	(5.62)	7.117	(5.10)	6.918	(5.47)
θ (current competition)	0.582	(3.90)	0.646	(4.64)	0.536	(3.62)	0.634	(4.55)	0.538	(3.64)	0.64	(4.60)
β (log (# next hour+1))	-5.057	-(4.58)	-3.542	-(3.61)	-6.755	-(6.61)	-5.783	-(6.35)	-6.577	-(6.41)	-5.522	-(6.05)
γ (log time until next)	2.617	(5.51)	3.232	(7.73)								
γ (same type next 5 auctions)					-7.441	-(4.89)	-8.172	-(6.07)				
γ (same type 1 a. from now)									-8.83	-(4.95)	-10.248	-(6.57)
γ (same type 2 a. from now)									-7.624	-(3.55)	-7.696	-(4.04)
γ (same type 3 a. from now)									-8.364	-(3.37)	-8.959	-(3.97)
γ (same type 4 a. from now)									-3.214	-(1.13)	-4.832	-(1.86)
γ (same type 5 a. from now)									-4.023	-(1.22)	-1.389	-(0.49)
	N=2317	$R^2=0.86$	N=2393	$R^2=0.88$	N=2372	$R^2=0.85$	N=2451	$R^2=0.87$	N=1693	$R^2=0.85$	N=1646	$R^2=0.87$

Discussion of the empirical findings

- Forward-seeing effects operate on eBay (3-7% price-reduction when the same type available within next 5 auctions, controlling for # bidders)
 - ⇒ Fairly high lower bound on bidder-sophistication
 - ⇒ Direction for specifying future more fine-grained structural models
 - ⇒ Analysts interested in demand-estimation should not interpret eBay auctions as repeated isolated auctions (downward bias)
- There may be forward-looking bid-shading beyond the reaction to already-listed “forward-seen” future auctions.
 - ⇒ Sellers may want to take note: such forward-looking bid-shading is a response to a seller strategy; bidding depends on selling and vice versa.
- Relevance beyond eBay: most sequences have look-ahead preannouncements...