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Matthew Panhans

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Matthew T. Panhans*

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Abstract

This article proposes a sequential model of a vertical supply chain and investigates the model's predictions about vertical integration. The model accommodates both linear and non-linear pricing, which is important because the amount of double margin distortion drives predictions about vertical integration. The upstream is modeled as a negotiation over wholesale prices, and the downstream is differentiated product price-setting. Existing literature has focused on modeling the upstream and downstream markets as simultaneously determining prices. A sequential model can yield significantly different predictions about the effects of vertical integration and readily accommodates linear and non-linear pricing in the upstream negotiations.

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1 Introduction

A sharp policy discussion concerns when to characterize vertical mergers as pro-competitive or anti-competitive. Because many incentives change with vertical integration, economic models will typically incorporate both efficient and anti-competitive effects. Recent policy debates regarding vertical mergers have centered on when each of these effects dominate, and how they should be treated under the antitrust laws. These debates reflect a heightened scrutiny of vertical mergers in recent years.[1]

The relative magnitudes of various pro- and anti-competitive effects in an economic model depend on the particular assumptions and modeling decisions. Much existing literature on vertical integration assumes simultaneous timing between the two levels of the supply chain and linear wholesale pricing. But these assumptions may not be appropriate in many settings, such as when wholesale contracts are long term and a sequential timing assumption would be more appropriate, or when pricing contracts deviate from linear pricing with two-part tariffs or volume discounting. It is not well understood how much these assumptions matter for predicted effects, and how much predictions can change with different assumptions. This paper advances our understanding of how certain assumptions in models of a vertical supply chain affect predictions about vertical integration, which is important for balancing potential pro- and anti-competitive effects of vertical integration.

Specifically, equilibrium analysis of vertical integration requires modeling competition at two levels of a supply chain, and also specifying how those two levels relate to each other. Much of the literature on vertical integration assumes a monopolist at one level of the supply chain, which greatly simplifies the interaction between the upstream and downstream markets and consequently facilitates the characterization of an equilibrium. Studies that allow for successive oligopoly markets with upstream bargaining and downstream price setting assume that the upstream bargaining occurs simultaneously with the downstream price setting, which also facilitates the equilibrium characterization (Draganska, Klapper, and Villas-Boas 2010; Sheu and Taragin 2021). However, in some settings it may be reasonable to posit that upstream wholesale prices are negotiated first, and downstream prices are subsequently set given the wholesale prices and the downstream competitive conditions. Such sequential timing may be desirable to model markets when wholesale contracts tend to be longer term and retail prices can change with greater frequency.[2]

[1]For example, the U.S DOJ and FTC have in recent years challenged non-horizontal mergers of AT&T/Time Warner, Illumina Inc./GRAIL, NVIDIA Corporation/Arm Ltd., Lockheed Martin Corporation/Aerojet Rocketdyne Holdings Inc., UnitedHealth/Change Healthcare, and Microsoft/Activision.

[2]Sequential timing may be appropriate in many industries. Retailers often have long-term supply contracts with wholesalers and can change retail prices with greater frequency. Crawford, Lee, et al. 2018, p. 911

Moreover, such sequential timing allows the model to readily incorporate non-linear pricing contracts, which impact the amount of double margin distortion in the market and thus have a large effect on a model's predictions about the effects of vertical integration. This paper proposes such a sequential model of successive oligopoly, shows how the equilibrium prices can be calculated, and then demonstrates how the model can be used to evaluate the effects of vertical integration.

Specifically, the model I propose involves a first stage where wholesalers and retailers negotiation over wholesale prices through Nash-in-Nash bargaining. In the second stage of the game, retailers compete in a price setting game and set retail prices to maximize profits taking the wholesale prices as given. An equilibrium is computed by solving the game through backward induction. Solving the model is complicated by the fact that closed form solutions are unavailable even with a standard logit demand system. I use numerical methods and an iterative algorithm to compute the model's equilibrium. I then show that a sequential timing assumption can predict greater harm, and sometimes substantially so, arising from vertical integration compared to a model with the same inputs but a simultaneous timing assumption. I also show that the sequential timing with a two-part tariff predicts greater harm than when contracts are based on linear prices.

Because different timing assumptions generate different strategic interactions, the same demand and costs will yield different equilibria and therefore different predicted merger effects in a sequential model than in models with simultanous timing. Indeed, I find that with the exact same demand parameters, bargaining weights, and marginal costs, a model with the sequential timing assumption can yield substantially different results on consumer welfare than a model with a simultaneous timing assumption.

A key advantage of a sequential model is that it lends itself to a general formulation that nests linear pricing and two-part tariffs as special cases. I embed the linear price and two part tariff contracts into a bargaining situation similarly to how Kourandi and Pinopoulos 2022 model the negotiation between an integrated firm and a downstream rival, but allowing for successively oligopoly. Since the elimination of double marginalization (EDM) can be an important effect to consider in evaluating vertical integration, it is useful to have a model that can account for contracts that eliminate at least some part of the double margin distortion in a vertical market without vertical integration. In contrast, a two-part tariff in a model with a simultaneous timing assumption does not yield a unique solution to the bargaining

describe that in multichannel video programming distribution, a sequential timing assumption might be more realistic than a simultaneous one. The negotiations in technology industries such as semiconductors over licensing fees and royalty rates might also lend itself to sequential timing (Vita et al. 2022).

problem. Because downstream prices are held fixed during upstream negotiations, there is no scope for negotiating agents to directly affect the downstream price through their bargaining. This means that if the surplus is being split by a lump-sum payment, then there is nothing to pin down the equilibrium wholesale prices in a model with the simultaneous timing assumption.

Having a model that can accommodate linear and non-linear pricing is especially important because the effects of vertical integration depend greatly on the amount of double margin distortion in the market. Numerical simulations of the model show that linear pricing and non-linear pricing can have very different implications for predicted merger effects. Moreover, previous literature shows that a wide variety of vertical contracts depart from simple linear pricing and have competitively important implications (Kühn 1997; Dana and Spier 2001; Villas-Boas 2007; Mortimer 2008; Conlon and Mortimer 2021). Thus having a model of a vertical supply chain that can account for linear and non-linear pricing is important for vertical merger evaluation.

Consistent with this, empirical evidence on vertical integration shows that the effects vary by institutional industry arrangements and contracting practices (Lafontaine and Slade 2007; Lafontaine and Slade 2021; Beck and Scott Morton 2021). Some of the studies that involve effects most related to the model in the current paper include Luco and Marshall 2020, who show that in the U.S. carbonated beverage industry, vertical integration of manufacturers and bottlers led to a decrease in prices for integrated products, but an increase in price for non-integrated products through what they call the Edgeworth-Salinger effect. The study also highlights the importance of modeling firms as multi-product firms in order to account for these various effects of vertical integration. Gray, Alpert, and Sood 2023 show evidence consistent with raising rivals costs and harm to consumers when evaluating vertical integration between an insurer and pharmacy benefit manager that occurred in 2015. Gil 2015 evaluates the effect of the 1948 Paramount antitrust case and finds that vertically integrated theaters charged lower prices and sold more admission tickets than nonintegrated theaters in 1945-1955, consistent with vertical integration lowering prices through the elimination of double marginalization. Crawford, Lee, et al. 2018 empirically estimate that there are offsetting effects from EDM and raising rival's costs from vertical integration of regional sports networks with programming distributors in U.S. television markets.

I carefully manipulate parameters of the model to show which incentives might be relevant for merger investigations. The equilibrium analysis shows that mergers in the linear pricing model tend to result in more consumer benefit than mergers in the non-linear pricing model, because there is more scope for EDM due to integration when negotiation is over a linear price contract. Similarly, having a greater market share of a product that becomes vertically integrated tends to create more increases in consumer surplus because there is a larger scope for EDM.

I also illustrate how one can implement the model by calibrating demand and cost parameters from observed market prices and shares, as one might during a merger investigation. Given the same prices and shares, I show how calibration and merger simulation yield different results under three different models. In one example market with symmetric firms and equal relative bargaining between upstream and downstream, a sequential model with a two-part tariff yields consumer harm, while a sequential model with linear prices yields some consumer benefit, and a simultaneous model yields the greatest consumer benefit from a vertical merger.

The primary disadvantage of the sequential timing assumption is that closed form expressions cannot be obtained. Thus, equilibria must be computed numerically. However, I show that the model does converge, and that different parameter starting values converge to the same result.

A large theoretical literature has considered the effects of vertical integration in models with linear pricing (Salinger 1988; Chen 2001; Ordover and Shaffer 2007). A number of papers investigate integrated firm behavior under both linear and non-linear pricing. For example, Moresi and Schwartz 2017 use a model with sequential timing to investigate an intergrated firm's incentive to expand or contract a downstream rival's output under linear pricing or a two part tariff. However, their analysis does not evaluate the effects when moving from no integration to vertical integration, as I do in this model.

Other papers consider the consequences of assumptions about timing for vertical mergers.[3] Rogerson 2021 proposes simple formulas for approximating the magnitude of an input price increase following a merger within a bargaining context, and for both sequential and simultaneous timing. Moresi 2020 evaluates a model where an upstream firm bargains with two downstream firms, with linear pricing. Moresi finds that vertical mergers in that context tend to lead to lower prices when there is a simultaneous timing assumption and higher prices when there is a sequential timing assumption. This is consistent with the results of the model in the present paper with successive oligopolies, as the simultaneous model tends to create a greater double margin distortion and thus leaves more scope for efficiency gains from EDM relative to a sequential model. Domnenko and Sibley 2023 consider a setting with an upstream monopolist, two downstream competitors, and sequential timing pre-merger.

[3] For a more general discussion of timing in vertical models with bargaining, see Lee, Whinston, and Yurukoglu 2021 p. 712-713.

Their model has upstream price setting rather than bargaining, and they allow for the timing of the model to change in the post-merger world to one where the integrated firm can choose to move first or second. Also, they consider only situations with linear prices, whereas my model allows for linear and non-linear contract structures.

Three papers on merger analysis most similar to the present one are Das Varma and De Stefano 2020, Sheu and Taragin 2021, and Bonnet, Bouamra-Mechemache, and Molina 2021, all of which only account for linear pricing in the upstream bargaining setting. Bonnet, Bouamra-Mechemache, and Molina 2021 use a sequential model similar to the one I use, though with a different informational assumption, to quantify the countervailing buyer power that can arise from downstream horizontal mergers.[4] Das Varma and De Stefano 2020 consider vertical integration in a setting with an upstream monopolist, two differentiated downstream competitors, upstream bargaining, and a sequential timing assumption. My model is a generalization of this model in that it allows for an upstream oligopoly and allows for non-linear pricing.[5] Sheu and Taragin 2021 has the same structure as the model in this paper, except with a simultaneous timing between the upstream and downstream markets.

This paper and Rey and Vergé 2020 are the first to pose models that consider the effects of vertical integration that can accommodate a range of pricing contracts and in successive oligopoly with upstream bargaining and downstream price setting. Having a model with the flexibility to account for different amounts of EDM from a vertical merger is an important tool for merger evaluation because (i) the amount of pre-merger double margin distortion is so critical to the predicted effects from vertical integration, and (ii) industries have been observed to have different contracting practices and informational structures. Rey and Vergé 2020 pose a more general theoretical model, with a general tariff structure and various extensions that allow the model to be used for analyzing RPM and pricing parity provisions, and resale vs. agency business models. My paper focuses on a readily implementable model for merger analysis and explores how varying inputs to that model change the model's predictions about the effects of vertical integration.[6]

[4] Bonnet, Bouamra-Mechemache, and Molina 2021 assume "interim unobservability", meaning that bargaining outcomes between manufacturers and a given retailer remain unobserved to other retailers during the downstream competition stage, while the present paper assumes public contracts.

[5] The results shown in the appendix table of their paper can be reproduced with the code I use to implement the model described in this paper, when the upstream is one firm with a single brand, and there are two downstream firms that each carry only the brand of the upstream firm.

[6] More specifically, my paper assumes a full information setting, while Rey and Vergé 2020 are primarily focused on settings with secret contracts. They do consider public contracting as an extension, and my simulations are consistent with their theoretical results for public contracting. Also there is a slight difference in the model specifications, where I have negotiating firms choose a two-part tariff that maximizes their joint

2 A Sequential Model with Bargaining

Suppose wholesalers own brands denoted by $w \in W$. For simplicity of notation, I will assume each wholesaler owns a single brand, so that w will denote both a single wholesale firm and a single brand, but the notation can easily be extended for wholesalers that own a portfolio of brands. Wholesalers distribute their brands through retailers, which are denoted by $r \in R$. A retailer can carry multiple brands. Each good, denoted by $j \in J$, is a brand-retailer combination. If all retailers carry all brands, there are $|J| = |W| \times |R|$ goods in the market. In the Nash-in-Nash approach used in this paper and in much previous literature, in equilibrium every link is formed and thus every brand is offered through every retailer.[7]

Pricing decisions occur sequentially, with a first stage in which wholesalers and retailers negotiation over wholesale prices, and a second stage in which retailers set retail prices. This can reflect that wholesale contracts tend to be longer term while retail prices can change with greater frequency.

In the first stage of the game, wholesalers and retailers negotiate over the wholesale prices p^W that retailers will pay wholesalers for each unit of goods supplied for the brands that they carry. This is modeled as occurring through Nash bargaining. Here, I do assume that all upstream negotiations occur simultaneously with each other, resulting in a "Nash-in-Nash" equilibrium upstream (Horn and Wolinsky 1988; Collard-Wexler, Gowrisankaran, and Lee 2019; Lee, Whinston, and Yurukoglu 2021).[8] Following the literature, firms in this model are assumed to hold passive beliefs for Nash-in-Nash (for details see Lee, Whinston, and Yurukoglu 2021, p. 687).

Once all upstream contracts are negotiated, retailers set downstream retail prices p^R in order to maximize their profits in a Nash Bertrand equilibrium, taking the wholesale prices as given and all upstream contracts as fixed and known.

Given retail prices, end consumers choose their most preferred options, and the market share for good j is denoted s_j. Assume there is an outside option, and for the baseline case, assume standard logit demand so that $s_j = \frac{e^{\delta_j - \alpha p_j^R}}{1+\sum_{k \in J} e^{\delta_k - \alpha p_k^R}}$ for each good.[9] For scenarios requiring

gains from trade, while Rey and Vergé 2020 have negotiating firms choose a general tariff structure that maximizes their joint profits.

[7]With differentiated goods, every bilateral negotiation generates some gains from trade, and there is always some split of that surplus that the wholesaler and the retailer can make and that will be mutually beneficial.

[8]The consequence of using alternative bargaining protocols, such as the Nash-in-Shapley equilibrium of Froeb, Mares, and Tschantz 2019, is an interesting area of ongoing research.

[9]That is, consumer i's utility for product j is given by $u_{ij} = \delta_j - \alpha p_j^R + \varepsilon_{ij}$, where ε_{ij} has the usual Gumbel or type I extreme value distribution.

more flexible substitution patterns, I specify the choice probability for each good as the generalized nested logit (GNL) demand. The market size is normalized to one.

2.1 Upstream Bargaining

Recall that W denotes the set of brands in the market. Define a subset of those brands which are carried by retailer r as W^r. Recall that each good j is a combination of brand and retailer, so each $j = (w, r)$ for some brand w and retailer r. When r and w are part of the same firm, I will say that good j is an *integrated good*, and otherwise good j is a *non-integrated good*. Then, further subset W^r into brands carried by retailer r that are integrated, and denote them by $\overline{W^r}$, and brands carried by r that are not integrated, and denoted by $\underline{W^r}$.

Similarly, let R^w denote the set of retailers in which w supplies its brand. Then $\overline{R^w}$ denotes the retailers that carry w and are integrated with the brand, while $\underline{R^w}$ denotes those retailers that carry w and are not integrated with brand w.

One can now specify the objective function for wholesaler w, who is maximizing profits:

$$\Pi^w = \sum_{t \in \underline{R^w}} (p^W_{tw} - c^W_{tw}) \cdot s_{tw} + \sum_{t \in \overline{R^w}} \Big(p^R_{tw} - c^R_{tw} - c^W_{tw}\Big) \cdot s_{tw} + \sum_{t \in r(w)} \sum_{v \in \underline{W^t}} \Big(p^R_{tv} - c^R_{tv} - p^W_{tv}\Big) \cdot s_{tv} \quad (1)$$

where p^R_{tw} denotes the retail price for brand w sold through retailer t, c^R_{tw} is the same product's constant marginal cost to the retailer, c^W_{tw} is the same product's constant marginal cost of production to the wholesaler, and p^W_{tw} is the wholesale unit price retailer t pays to brand w.

The first summation in the profit function accounts for products brand w sells through independent retailers, and on which it obtains the wholesale margin. If the brand is not integrated with any retailers, this summation accounts for the entire firm profits. The second and third summations are non-zero only if the brand is integrated with some retailers. The second summation capture the profits on the sales of integrated goods, meaning brand w sold through a downstream retail division that is jointly owned. On these products, the firm captures the full downstream and upstream margin. The third summation accounts for non-integrated goods that are sold through downstream divisions of the same firm that owns brand w. Define $r(w)$ as the set of retailers that are jointly owned by the same firm that owns brand w, and so the double summation accounts for profits from non-integrated goods sold by retailers that jointly own w.

Specify an analogous objective function for a retailer r:

$$\Pi^r = \sum_{v \in \underline{W^r}} \Big(p^R_{rv} - c^R_{rv} - p^W_{rv} \Big) \cdot s_{rv} + \sum_{v \in \overline{W^r}} \Big(p^R_{rv} - c^R_{rv} - c^W_{rv} \Big) \cdot s_{rv} + \sum_{v \in w(r)} \sum_{t \in \underline{R^v}} \Big(p^W_{tv} - c^W_{tv} \Big) \cdot s_{tv} \quad (2)$$

where the first term accounts for profits on non-integrated goods sold by the retailer, the second summation accounts for profits on integrated goods sold by the retailer, and the third summation accounts for the wholesale margin earned on non-integrated goods sold by upstream wholesale divisions jointly owned with retailer r. If the retailer r has no integration with any upstream wholesale divisions, then only the first summation in the profit function is non-zero.

Finally, I need to define disagreement payoffs. Consider a good j which is brand w sold through retailer r. If w and r fail to reach an agreement, the share for the good would fall to zero and the share of the other goods would increase. Denote the disagreement share of good k when good j is no longer supplied in the market by $\tilde{s_k}(j)$. Then when good j is no longer supplied in the market, $\tilde{s_j}(j) = 0$ and $\tilde{s_k}(j) > s_k$ for all $k \neq j$. Disagreement profits for the wholesaler and retailer when j is no longer supplied can be denoted by $\widetilde{\Pi^w}(j)$ and $\widetilde{\Pi^r}(j)$, respectively. In disagreement, I hold other wholesale prices fixed, but allow the downstream prices to adjust.[10]

Linear prices

We now have all of the pieces to define the upstream equilibrium under linear pricing, meaning there are no lump-sum payments allowed between negotiating retailers and wholesalers. In this case, equilibrium wholesale prices are defined as those that maximize the Nash product, which is given by:

$$\max_{p^W_j} \left(\Pi^r - \widetilde{\Pi^r}(j) \right)^{\lambda} \left(\Pi^w - \widetilde{\Pi^w}(j) \right)^{1-\lambda} \qquad \forall j \in J \quad (3)$$

[10] Sheu and Taragin 2021 hold both the wholesale and retail prices fixed in disagreement, as the upstream and downstream equilibria are simultaneously achieved in that model. With the sequential timing assumption, however, the downstream retail prices should be allowed to adjust in the disagreement scenarios. Crawford and Yurukoglu 2012 and Das Varma and De Stefano 2020 use sequential models and also allow the downstream prices to adjustment in disagreement while holding the wholesale prices fixed.

where λ denotes the relative bargaining ability of retailers vis a vis wholesalers. For simplicity, assume the bargaining ability is constant across all retailer-wholesaler pairs.

Two part tariff

In some situations, it may be reasonable to think that in addition to the wholesale price that is negotiated, retailers and wholesalers may be able to also negotiate a lump sum payment as part of their negotiated contract. Denote lump sum payments by F. When r and w negotiate over the good j, the Nash product in this situation becomes:

$$\max_{p_j^W, F} \left(\Pi^r - \widetilde{\Pi^r}(j) - F\right)^{\lambda} \left(\Pi^w - \widetilde{\Pi^w}(j) + F\right)^{1-\lambda} \qquad \forall j \in J \tag{4}$$

Solving this equation for F, and then substituting the expression for the optimal F back into the Nash product, yields an expression that is proportional to the joint surplus $(\Pi^r - \widetilde{\Pi^r} + \Pi^w - \widetilde{\Pi^w})$, as shown in Appendix A. What this means is that with a two part tariff, the wholesale price is negotiated such that it maximizes the joint profits among the negotiating parties, and then the lump sum payment F is set so that the total available surplus will be split according to the bargaining parameter λ.

Because the negotiating agents are able to influence the downstream price through their negotiated wholesale price, they have more flexibility under a two part tariff to maximize an objective function that does not depend on the bargaining parameter. Note also that in a simultaneous setting such as Sheu and Taragin 2021, where upstream and downstream competition occurs simultaneously, it is not straightforward to incorporate a two part tariff contract in this way; because downstream prices are held fixed during upstream negotiation, there is no scope for negotiating agents to affect the downstream price through their bargaining. Thus, in a simultaneous model, changing p^W will not change the joint profit $(\Pi^r + \Pi^w)$ because p^R are held fixed. See Appendix A for further discussion on the two-part tariff with simultaneous timing.

In a model of sequential price setting, a linear price and a two part tariff contract will yield different equilibrium wholesale and retail prices. And moreover, the two types of contracts can be combined into a single objective function with a weighting parameter. Suppose there is a market with imperfect contracting, so that negotiating firms are able to imperfectly contract for a lump sum payment. Let σ indicate the exogenous degree to which contracting is feasible, with $\sigma = 0$ indicating that only linear pricing is possible, $\sigma = 1$ indicating that a lump sum payment can be made that can fully transfer the available surplus. Any value of

$\sigma \in (0,1)$ can then indicate the degree to which contracting achieves efficient negotiations.[11] The general upstream objective function that nests the linear price and two part tariff scenarios can then be expressed as:

$$\max_{p_j^W} \quad (1-\sigma)\left(\Pi^r - \widetilde{\Pi^r}(j)\right)^{\lambda}\left(\Pi^w - \widetilde{\Pi^w}(j)\right)^{1-\lambda} + \sigma \cdot L \cdot \left(\Pi^r - \widetilde{\Pi^r}(j) + \Pi^w - \widetilde{\Pi^w}(j)\right)\Bigg(\quad \forall j \in J \tag{5}$$

where $L = \lambda^{\lambda} \cdot (1-\lambda)^{1-\lambda}$.

2.2 Downstream Retail Price Setting

In the second stage of the game, retailers take wholesale prices p^W as given, and then set downstream prices p^R of goods they carry in order to maximize their profits given in Equation 2. The model of competition is differentiated product price setting and the solution characterized by Nash Bertrand equilibrium. The profit function for retailers is as given above in Equation 2, and taking the derivatives with respect to p_j^R for each good j yields the first order conditions used to solve for equilibrium downstream prices.

2.3 Computing an Equilibrium

One disadvantage of the sequential model is that it is difficult to obtain a closed form solution for the equilibrium prices. In particular, closed form expressions require knowning how downstream demand changes with respect to the wholesale prices, i.e., $\frac{\partial s_j}{\partial p_j^W}$. When the upstream and downstream equilibria are computed simultaneous, this derivative is always equal to zero by assumption, and closed form first-order conditions are more straightforward to obtain. When the equilibria are computed sequentially, this derivative is not equal to zero.

Thus, in order to find equilibria in this model in a tractable way, I numerically maximize

[11] Most of this paper considers the polar cases of linear pricing ($\sigma = 0$) and two-part tariff ($\sigma = 1$). However, a model with intermediate values of $\sigma \in (0,1)$ may be useful in some cases. Gayle 2013 finds empirical patterns suggesting that airline codeshare contracts do not eliminate double margin distortions when the operating carrier of a codeshare product also offers competing products in the market. Jeuland and Shugan 1983 and Moorthy 1987 show that an optimal schedule of quantity discounts can eliminate double marginalization. Quantity discount schedules that are not exactly optimal due to imperfect information may thus lead to a partial but incomplete reduction in double marginalization. Kwoka and Slade 2020, p. 53 suggest that "It might be argued that the contracting alternative might itself be costly and cannot fully achieve the benefits of EDM... Where incomplete internalization can be documented, the proper antitrust calculus would be to credit integration with no more than the incremental cost savings over and above what contracting can achieve."

the upstream objective functions. The algorithm described here allows for the numerical computation of equilibrium prices.

Suppose the structural demand parameters, upstream and downstream marginal costs, bargaining parameter (λ), and extent of linear pricing (σ) are known. Then the equilibrium can be computed by the following algorithm:

1. Specify a tolerance level `tol` and initial guess of wholesale prices $p^{W,0}$
2. For each good $j = 1, 2, ..., J$, one by one, update $p_j^{W,0}$ to $p_j^{W,1}$, which is the value that numerically maximizes the Nash product for j given all the other wholesale prices fixed. Note that this also involves computing new downstream prices p^R each time a wholesale price changes.
3. After looping through each good, compute the change $C = \max(|p^{W,1} - p^{W,0}|)$. If $C >$ `tol`, go back to the previous step. If $C \leq$ `tol`, end and assign the current wholesale prices as the equilibrium values $p^{W,*}$.

While I have not proven the existence or uniqueness of the resulting equilibrium, numerical simulations show that this algorithm converges, and that for various starting values, the algorithm converges to the same result. This is consistent with the theoretical results provided by Rey and Vergé 2020 that an equilibrium exists and is unique in a sequential model with upstream bargaining, downstream price-setting, and a two-part tariff. The authors show that with a general tariff structure, sufficiently concave tariffs generate convex profit functions that make ensuring the existence of a Nash equilibrium problematic. They also show that restricting the tariff structure to two-part tariffs avoids the convexity issue, and ensures that an equilibrium exists in this model as long as an equilibrium exists in the downstream price-setting game for any cost profile.

3 Vertical Integration and Downstream Incentives

In order to investigate the effects of vertical integration in the market, I will first focus only on the downstream equilibrium while holding wholesale prices fixed. Though these results have been discussed in previous literature, it helps to isolate which effects of vertical integration occur due to changes in bargaining leverage from those due to changes in unilateral downstream pricing incentives. In the next section I will allow for a full equilibrium involving both the upstream and downstream markets.

There are two types of brands that a retailer might carry: integrated goods where the brand is jointly owned by the retailer's upstream divisions, and non-integrated goods whose brands are not jointly owned by the retailer.

When good $j = (r, w)$ carried by retailer r is a non-integrated good, the first-order condition pricing equation for the good coming from differentiating Equation 2 is:

$$
\begin{aligned}
\frac{\partial \Pi^r}{\partial p^R_{rw}} = & \sum_{v \in \underline{W^r} \setminus w} \Big((p^R_{rv} - c^R_{rv} - p^W_{rv}) \cdot \frac{\partial s_{rv}}{\partial p^R_{rw}} + (p^R_{rw} - c^R_{rw} - p^W_{rw}) \cdot \frac{\partial s_{rw}}{\partial p^R_{rw}} + s_{rw} \\
& + \sum_{v \in \overline{W^r}} \Big(p^R_{rv} - c^R_{rv} - c^W_{rv}) \cdot \frac{\partial s_{rv}}{\partial p^R_{rw}} \\
& + \sum_{v \in w(r)} \sum_{t \in \underline{R^v}} \Big(p^W_{tv} - c^W_{tv}) \cdot \frac{\partial s_{tv}}{\partial p^R_{rw}} = 0
\end{aligned} \tag{6}
$$

The first line captures profits from non-integrated goods sold by the retailer; if the retailer is not vertically integrated, the FOC reduces to only this first line, and is equivalent to Equation (2) in Sheu and Taragin 2021. When the retailer is the downstream division of a firm that also owns brands, the pricing incentive also depends on integrated goods sold at that retailer, which is captured in the second summation in the FOC, and sales of co-owned brands sold through rival retailers, which is captured in the final line of the FOC.

This FOC can be used to investigate the change in incentive on a retailer's pricing of non-integrated goods after vertical integration. Note that if a retailer is vertically integrated, a good that it sells that is non-integrated is by definition a rival brand to its upstream division. This FOC captures pricing incentives for those types of goods. Because the retailer carries some integrated goods after vertically integrating, it internalizes the double margin distortion on those goods. This FOC captures the incentive of a retailer to raise prices on its non-integrated goods to shift sales to its more profitable integrated goods: where s_{rv} is the share of an integrated good, $\frac{\partial s_{rv}}{\partial p^R_{rw}} > 0$. Thus, even when holding wholesale prices fixed, if there is some elimination of a double margin distortion, a retailer carrying integrated goods has some incentive to raise prices on rival brands that it carries. This effect has been called the "Edgeworth-Salinger effect" by Luco and Marshall 2020. Similarly, the final line of the FOC captures that when the retailer raises the price of non-integrated goods, some sales will be recaptured by its co-owned brands at other retailers. This too gives the retailer an incentives that it did not have before to increase the price of rival brands that it carries.

When good $j = (r, w)$ carried by retailer r is an integrated good, the first-order condition pricing equation for the good coming from differentiating Equation 2 is:

$$
\begin{aligned}
\frac{\partial \Pi^r}{\partial p^R_{rw}} = &\sum_{v \in \underline{W^r}} (p^R_{rv} - c^R_{rv} - p^W_{rv}) \cdot \frac{\partial s_{rv}}{\partial p^R_{rw}} + \sum_{v \in \overline{W^r} \setminus w} (p^R_{rv} - c^R_{rv} - c^W_{rv}) \cdot \frac{\partial s_{rv}}{\partial p^R_{rw}} \\
&+ (p^R_{rw} - c^R_{rw} - c^W_{rw}) \cdot \frac{\partial s_{rw}}{\partial p^R_{rw}} + s_{rw} \\
&+ \sum_{v \in w(r)} \sum_{t \in \underline{R^v}} (p^W_{tv} - c^W_{tv}) \cdot \frac{\partial s_{tv}}{\partial p^R_{rw}} = 0
\end{aligned}
\tag{7}
$$

The first summation captures profits from non-integrated goods sold by the retailer. The second summation captures profits from integrated goods sold by the retailer other than good $j = (r, w)$, the terms on the second line capture good $j = (r, w)$, and the final summation captures profits on brands that are owned by an upstream division that is jointly owned with retailer r, but that are sold through rival retailers.[12]

After vertically integrating good $j = (r, w)$, that good shifts from being in the set of non-integrated brands $\underline{W^r}$ to integrated brands $\overline{W^r}$. The first impact of this shift is that the margin that the retailer accounts for from sales of that good changes from the retail margin only $(p^R_{rw} - c^R_{rw} - p^W_{rw})$ to the full retail and wholesale margin $(p^R_{rw} - c^R_{rw} - c^W_{rw})$. This EDM effect allows the firm to internalize pricing and increase profits by lowering the retail price and expanding sales of that product after its integration. There is an offsetting effect, however, which is that after integration the retailer also accounts for how the retail price of an integrated good affects sales of co-owned brands at rival retailers. Since such sales were previously unprofitable to the retailer, but now it does capture the upstream/wholesale margin on co-owned brands that are sold through rival retailers, there is some recapture effect.[13] This puts upward pressure on the retail price of the integrated good. If the wholesale margin of brands sold through rival retailers is high enough $(p^W_{t,v(r)} - c^W_{t,v(r)})$, and if the diversion from the integrated good is high enough $(\frac{\partial s_{t,v(r)}}{\partial p^R_{rw}})$, this recapture effect can be greater than the EDM effect, and retail prices in the market can rise after vertical integration of good j, holding all wholesale prices fixed.

[12]This equation is equivalent to Equation (9) in Sheu and Taragin 2021 though with slightly different notation.

[13]Sheu and Taragin 2021 call this the "Wholesale UPP Effect."

4 Equilibrium Effects of Vertical Integration

In this section I now allow wholesale prices to adjust. Given demand parameters, costs, and the model of competitive interactions specified above, I compute an equilibrium without vertical integration, and an equilibrium with vertical integration, and show the equilibrium effects in various scenarios.

Vertical integration has several effects on negotiations, depending on which agents are negotiating. Suppose wholesaler w and retailer r integrated vertically, so that good $j = (r, w)$ becomes an integrated good. I first consider how integrating affects the negotiation between the upstream division w of the integrated firm with an independent downstream retailer, denoted t. Next, I will consider how integration affects the negotiation between the downstream division r of an integrated firm with an independent wholesaler, denoted by v.

When w is negotiating with an independent retailer t, it has a different profit function after it vertically integrates. In particular, for good j it previously received only the wholesale margin on each sale, while after integration it also receives the retail margin. In the notation in Equation 1, the good moves from the set of non-integrated goods $\underline{R^w}$ where it receives only the wholesale margin, to the set of integrated goods where it receives the full product margin $\overline{R^w}$. When in disagreement with the independent retailer t, the good $x = (t, w)$ is no longer supplied in the market, and the integrated good receives more sales than in the observed equilibrium, $\tilde{s_j}(x) > s_j$. After integration, the profitability of those captured sales increases because they are now to an integrated good for which the firm internalizes the full channel margin (retail margin + wholesale margin). Thus, the integrated firm has more bargaining leverage vis a vis independent retailers than it did before integrating. This additional leverage will allow the integrated entity to obtain a higher wholesale price for goods is sells through retailers it negotiates with.

A similar logic applies to non-integrated goods that are sold through w's downstream division; without integration, when w ended in disagreement with an independent retailer, it would not recapture any sales that diverted to r's non-integrated goods. With integration, w does account for those sales, giving it greater leverage in the negotiations with independent retailers and thus allowing it to obtain higher wholesale prices. These two effects can be called "raising rival retailer cost" incentives and put upward pressure on the wholesale price of w's goods sold through rival retailers.

When r is negotiating with a rival wholesaler v, an analogous logic applies that give it greater leverage. In disagreement with rival wholesaler's brand v, r now accounts for the fact that sales that would be recaptured through integrated good j will give it the full product margin, rather than only the retail margin. And secondly, r now accounts for the fact that in disagreement, meaning it no longer carries brand v supplied by a rival wholesaler, some sales from that good would divert to w's brands at other retailers. Both of these effects mean that r will be able to obtain lower wholesale prices when negotiating with rival wholesalers than it could without integration. These effects put downward pressure on the wholesale prices of goods sold through the downstream retail division r.

The strength of all of these various effects depends on how close of substitutes different products are, which can be captured in diversion ratios, and the corresponding profit margins. I use merger simulations in a simple setting to illustrate how various forces can affect retail prices of integrated and non-integrated goods after vertical integration occurs.

4.1 Setup for Simulations

For the baseline scenarios, consider a market with four goods, two brands, and two retailers. The market has full linkages, meaning that both retailers carry both of the brands. Pre-merger, every firm is independently owned and there is no vertical integration.

Wholesaler	w_1	w_2	w_1	w_2
Retailer	r_1	r_1	r_2	r_2
Good	j_1	j_2	j_3	j_4

Table 1: Pre-merger industry structure

$$j_1 = (r_1, w_1),\ j_2 = (r_1, w_2),\ j_3 = (r_2, w_1),\ j_4 = (r_2, w_2)$$

For merger simulations, consider that r_1 and w_1 integrate, so the post-merger market structure becomes:

Wholesaler	w_1	w_2	w_1	w_2
Retailer	w_1	w_1	r_2	r_2
Good	j_1	j_2	j_3	j_4

Table 2: Post-merger industry structure

$$j_1 = (w_1, w_1),\ j_2 = (w_1, w_2),\ j_3 = (r_2, w_1),\ j_4 = (r_2, w_2)$$

In the baseline scenarios, I also assume that demand is given by a standard logit demand system, where each good has an identical mean value term $\delta_j = 3.5$, and there is a common price coefficient $\alpha = 1.2$. Retail costs are constant across goods at $c^R = 0.2$ and wholesale costs are $c^W = 0.2$. Unless otherwise specified, the baseline also assumes linear pricing ($\sigma = 0$) and equal relative bargaining abilities ($\lambda = 0.5$).

For most cases, the outcome reported is the change in consumer surplus after vertical integration occurs. Given the assumption of the logit demand system, the change in consumer surplus is calculated simply as the change in the log sum or "inclusive value" term multiplied by the marginal utility of income. Specifically, for the logit system, the change in consumer surplus that is reported is given by $\Delta CS = \frac{1}{\alpha} \cdot \left(\log(\sum_{j \in J} e^{\delta_j - \alpha p_j^1}) - \log(\sum_{j \in J} e^{\delta_j - \alpha p_j^0}) \right)$, where p^1 indicates post-merger prices and p^0 indicates pre-merger prices.[14]

4.2 Linear pricing and two part tariffs

First consider the equilibrium effects on consumer welfare of vertical integration when upstream price negotiations are based on linear pricing, compared to relaxing that assumption. Figure 1 shows the effects on consumer surplus as the pricing policy goes from linear pricing only ($\sigma = 0$) to a full ability to contract through a two part tariff ($\sigma = 1$). As expected, vertical integration has the greatest benefit to consumer surplus under linear pricing. Because linear pricing creates a large pre-merger double margin distortion, there is a large scope for the EDM effect to be large in this scenario. As the pricing policy moves toward a two part tariff, the relative benefit to consumer surplus of vertical integration falls.

Table 4 shows the underlying prices and shares for each good in each scenario. As the scenarios move from linear prices ($\sigma = 0$) toward a fully implementable two-part tariff ($\sigma = 1$) in the market, the pre-merger equilibrium wholesale prices fall, moving toward the wholesale costs for every good, as one might expect based on theory that two-part tariffs can achieve more efficient outcomes.

However, the wholesale prices never reach the wholesale costs, even with the full two-part tariff. Previous theoretical literature has shown that with secret contracts (i.e. where the tariff is private information between the two negotiating parties), equilibrium tariffs are cost-based (O'Brien and Shaffer 1992; Rey and Vergé 2020). With public contracting, which corresponds to the full-information setting I use in this paper, Rey and Vergé 2020 show that equilibrium two-part tariffs will yield a marginal unit wholesale price greater than production costs ($p^W > c^W$), because the public contract provides a strategic incentive to raise wholesale prices to induce other retailers to raise their retail prices. This theoretical point is consistent with the results of the simulations shown here.

[14]See Train 2009 p. 65 and p. 95.

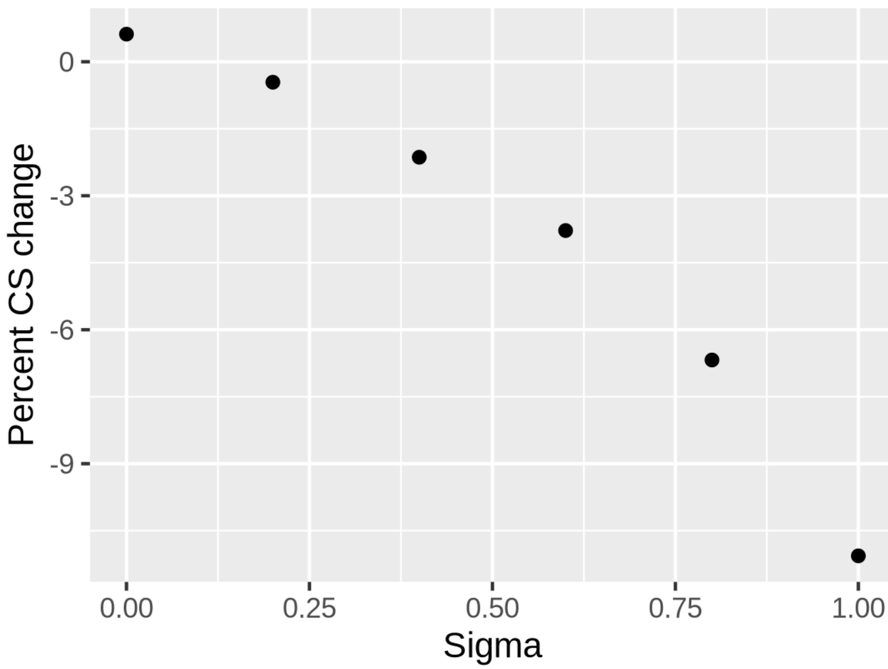

Figure 1: The benefits to consumer surplus of vertical integration are greatest under linear pricing. $\sigma = 0$ corresponds to linear pricing and $\sigma = 1$ corresponds to a two part tariff.

Interestingly, the pre-merger equilibrium wholesale prices smoothly move toward the wholesale cost values as the contracting environment moves from linear pricing toward full two-part tariffs. Post-merger, however, the integrated firm's increased bargaining leverage gives it an incentive to move further away from the efficient wholesale prices when dealing with rival downstream firms. This is a key mechanism of harm in this model, sometimes referred to as a "Bargaining raising rival's costs (RRC)" effect.

The following scenarios focus on the pure linear pricing case ($\sigma = 0$) and the full two part tariff ($\sigma = 1$) case, rather than the intermediate cases. But it is noteworthy that the model has the flexibility to account for intermediate situations and that there is a smooth change in consumer welfare effects when going from other case to the other.

4.3 Market shares

Next consider how the diversions among products in the market affects vertical integration.

I first consider how changing the size of the outside option affects the equilibrium merger effects. I do so by scaling up and down the mean value parameter δ for all goods simultaneously. Recall that in the baseline, the δ_j is the same value for all products. I maintain that assumption, but scale up and down the mean values together, which mechanically changes the size of the outside option. The standard logit demand assumed here causes the scaling up of the outside option to decrease the diversion ratios among goods in the market. As the outside good increases in size, the logit assumption implies greater diversion to the outside good, and thus lower diversions among products in the market.

Figure 2 shows that as the size of the outside option increases, and thus diversion ratios among the products decrease, the merger is more beneficial to consumer welfare. In other words, when the size of the outside option is small, and thus diversion ratios among the products are high, there is more scope for the change in incentives from integration to lower consumer welfare.

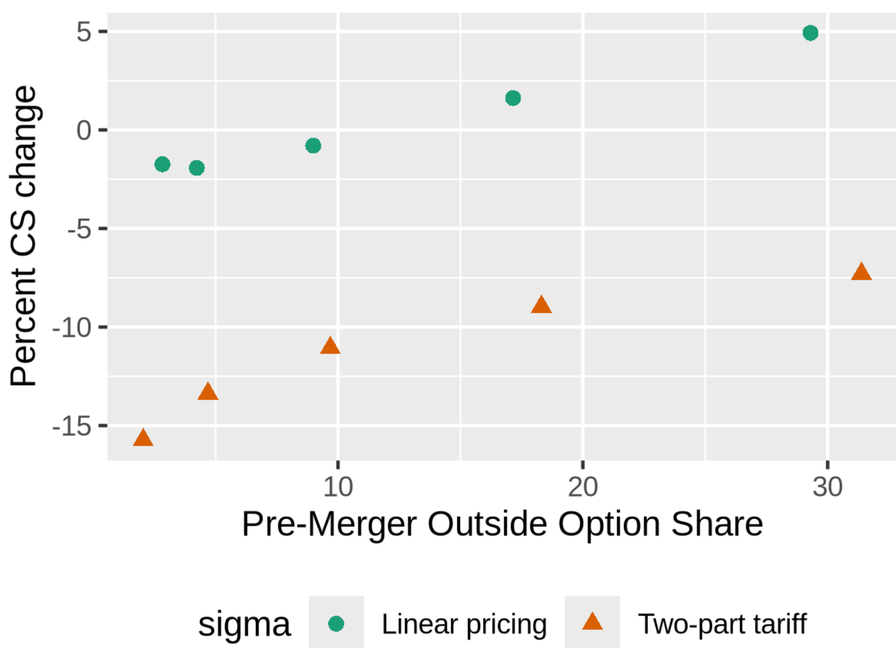

Figure 2: Increasing the size of the outside options lowers diversions among the products, and tends to make integration more beneficial. The efficiency effects get stronger over harmful effects on consumer surplus.

Next, I investigate the importance of the trade between integrating firms, relative to other trading partners. I increase the mean value δ_1 where $j = 1$ is the good that becomes vertically integrated, and keep all other mean values fixed at the baseline values. Increasing δ_j for the integrating good has the effect of increasing the pre-merger sales of the good that becomes vertically integrated, holding all else fixed.

Figure 3 shows that increasing the pre-merger size of the good that becomes vertically integrated makes the integration more beneficial to consumers under linear pricing. This is because under linear pricing, the double margin distortion is relatively large. Having a larger amount of the market become integrated allows for a greater amount of EDM after integration. With linear prices, there is more scope for EDM as the pre-merger size of the good that becomes integrated increases. However, this is not the case with the two-part tariff. Because the double margin distortion can be addressed through contracting, vertical integration in this scenario does not have as much scope to generate significant EDM. In fact, integration in the scenario specified here when there is a two-part tariff always harms consumers, and increasing the market share of the integrated good can make the merger more harmful to consumers.

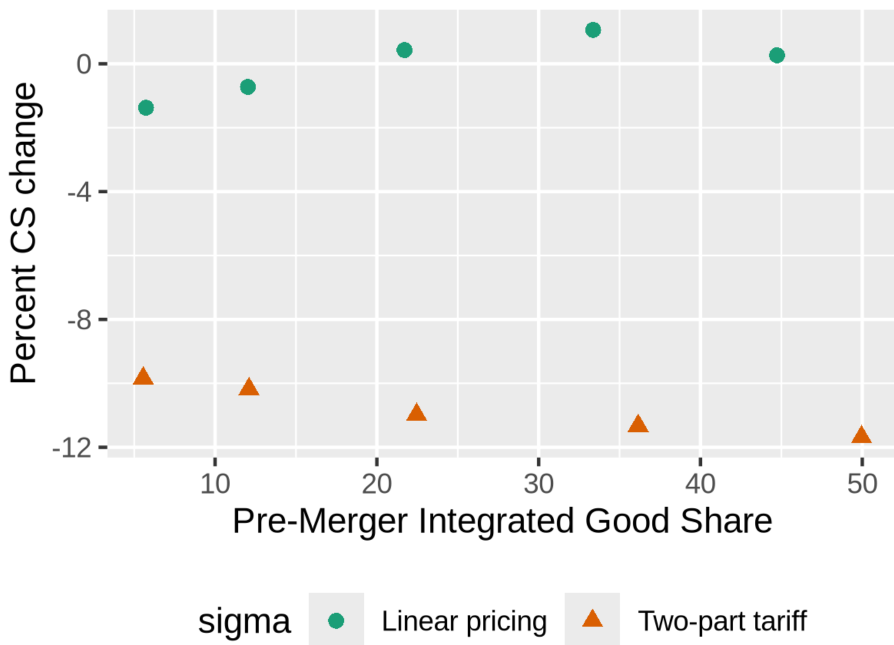

Figure 3: Increasing the pre-merger size of the good that becomes vertically integrated makes the integration more beneficial to consumers under linear pricing and more harmful to consumer under the two-part tariff.

4.4 Comparisons to a simultaneous equilibrium model

Assuming a sequential timing assumption where the downstream retail prices are set after the upstream wholesale prices are determined creates different strategic interactions than in a model with a simultaneous timing assumption. Even with the same demand and cost parameters and bargaining weights, the resulting equilibrium will be different and thus the effects of vertical integration on consumer welfare will be different.

Figure 4 shows the results of a merger simulation in the sequential model compared to the results of a merger simulation based on the exact same demand and cost parameters but in the Sheu and Taragin 2021 simultaneous model. Moving along the x-axis shows how the differences vary with the bargaining weight. The figure shows that a sequential model can yield substantially different predictions for consumer surplus than a simultaneous model with the exact same demand and cost parameters. The underlying pre and post merger values are shown in Table 7.

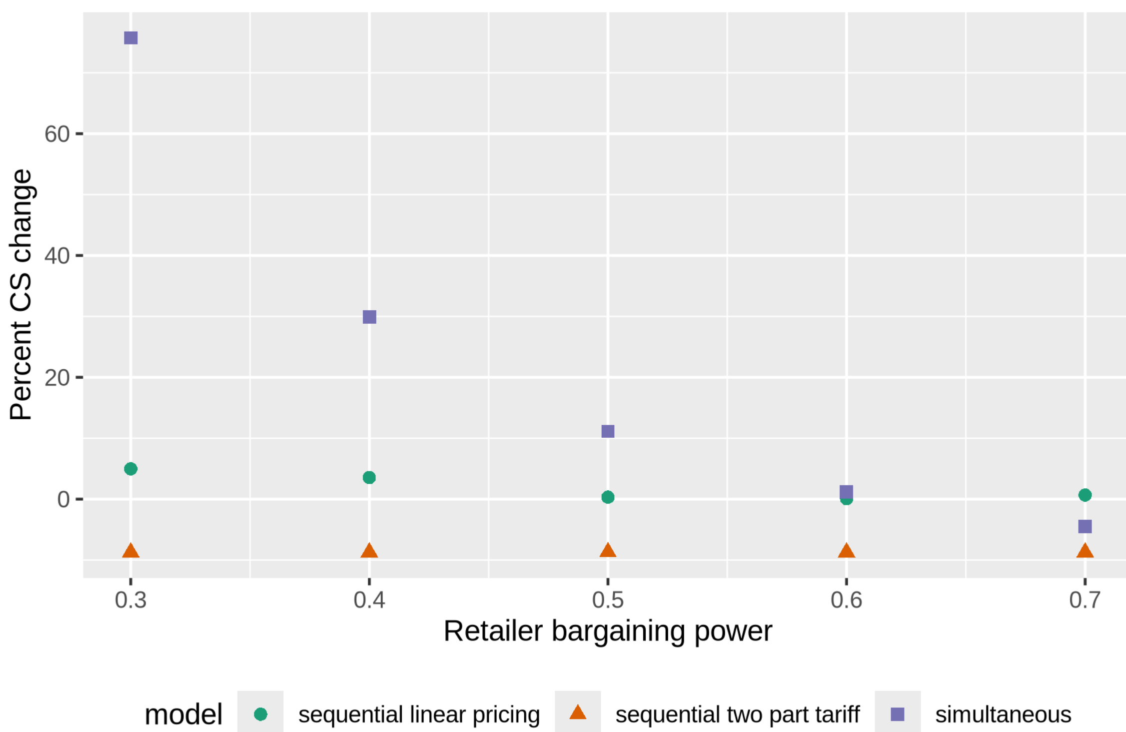

Figure 4: A sequential model can yield substantially different predictions for consumer surplus than a simultaneous equilibrium model with the exact same demand and cost parameters.

One clear pattern from this exercise is that in the simultaneous model, the bargaining parameter is a primary driver of the predicted effects of integration on consumer welfare. This is consistent with the pattern shown in figure 6 of Sheu and Taragin 2021. In this model, the relative bargaining power determines how much double margin distortion exists in the pre-merger world. When upstream wholesalers have a lot of bargaining power (and thus the value of λ is close to zero), there is a large double margin distortion, and integration generates large benefits. When wholesalers have little bargaining power (and thus the value of λ is close to one), the wholesale price approaches the wholesale costs and there is little distortion and integration causes fewer benefits and more harm.

In the sequential model, the bargaining parameter has a less drastic impact on the predicted effects of a vertical merger. In the case with a two-part tariff, the bargaining parameter has no effect at all. This is because with a full two-part tariff, the lump sum payment is used to split the surplus, and thus wholesale prices are determined completely independently of the value of the bargaining parameter.

The more interesting comparison with the simultaneous model is to linear pricing in the sequential model, where the bargaining parameter does have an effect. But the pattern is different in the sequential model compared to the simultaneous model. In the scenario shown here, consumer surplus has a slight U-shape. Mergers with sequential timing and linear prices at these parameter values generate the most benefit when bargaining power is asymmetric, and the least benefit when the relative bargaining between retailers and wholesalers is more even.

In order to delve more deeply into what drives the differences in predictions between the two linear price models that differ only in the timing assumption, I show in Table 8 the product-level prices and shares underlying Figure 4. One reason for the different predictions seems to be that the simultaneous model tends to have greater distortions from double marginalization than the sequential model. With the same costs and demand parameters, the simultaneous model always has higher pre-merger retail and wholesale prices than the sequential model. As a consequence, the integrated good post-merger almost always has a greater decline in its price with the simultaneous timing, and sometimes significantly so.

It makes intuitive sense that the scope for double marginalization is greater in the simultaneous model, where the upstream negotiations occur given a fixed downstream price. In equilibrium those upstream prices need to be optimal given the downstream prices, and vice versa, but there is less scope for the upstream negotiators to influence the downstream price. In the sequential model, the upstream negotiations occur knowing that downstream retail prices will be set taking as given the results of the upstream negotiation. This gives the negotiating agents a greater ability to influence what the downstream prices will be, and thus an opportunity to better ameliorate the amount of distortion. As a consequence of the greater double marginalization with the simultaneous timing, the model often predicts greater efficiencies from integration relative to the sequential model, particularly when retailers have less bargaining power.

Other effects also vary with the bargaining parameter, and this means that the simultaneous and sequential timing can have different overall predicted effects depending on the bargaining parameter. For example, all models at these parameter values have the merged entity increase the price of good 2 post-merger (this is the good that is owned by a rival brand but sold at the integrated retailer). Because the integrated good becomes more profitable to sell, it makes sense that the merged firm would increase the price of the rival brand sold at the integrated retailer in order to shift some sales to the integrated good. However, the strength of this effect is different in the simultaneous and sequential models. In the simultaneous model, the percentage price increase for good 2 is roughly similar for any value of the bargaining parameter. In the sequential model, the percentage price increase on good 2 is greatest when there is equal bargaining power between retailers and wholesalers. That the various effects net out differently depending on the timing assumption is part of the explanation as to why the simultaneous model shows a monotone decrease in the percent change in consumer surplus as the bargaining power increases, while the sequential model shows more of a U-shaped effect on consumer surplus.

4.5 Importance of brands vs retail banner

An advantage of numerically solving this model is that it can be readily extended to more general demand systems, requiring only that the choice probability function be correctly specified. Consider a more general demand structure based on the generalized nested logit (GNL) demand model described in Chapter 4 of Train 2009. The GNL model captures important possible demand patterns that standard logit and nested logit cannot capture. Because in this setting each good is a retailer-brand combination, the use of only a nested logit structure requires making a difficult decision of whether nests should be based on brands or retailers. The GNL demand framework allows for overlapping nests, and so in this setting, each product can belong to both the nest for its retailer and the nest for its brand/wholesaler.

Consider a set of K nests denoted $B_1, B_2, ..., B_K$. Each good j belongs to one or more nests. An allocation parameter a_{jk} designates the degree of membership of good j in nest k. For interpretation, let $\sum_k a_{jk} = 1$ for each good j. Lastly, $\mu_k \in [0, 1]$ is the nesting parameter for nest k. Following Train 2009, the probability that alternative j is chosen is then given by:

$$s_j = \frac{\sum_k (a_{jk} \cdot e^{V_j})^{1/\mu_k} \cdot \left(\sum_{i \in B_k} (a_{ik} \cdot e^{V_i})^{1/\mu_k}\right)^{\mu_k - 1}}{\sum_{l=1}^{K} \left(\sum_{i \in B_l} (a_{il} \cdot e^{V_i})^{1/\mu_l}\right)^{\mu_l}}$$

If each good is in exactly one nest, so that $a_{jk} = 1$ for nest k and $a_{jk'} = 0$ for all other nests k', this model becomes a nested logit. If in addition, $\mu_k = 1$ for all nests, then the model becomes standard logit.

This framework for demand allows for goods to belong to both a brand nest and a retailer nest, and for consumers to value particular brands and retailers more than others. Different strengths of brand and retailer recognition to consumers will affect the contours of harm or benefit that result from vertical integration in a market.

Figure 5 shows the effects of a vertical merger for a range of nesting structures within the GNL structure and linear pricing. In all cases, there are nests for each brand, and there are nests for each retailer, and each good is assigned to two nests: one for its brand and one for its retailer. The nesting parameter for every nest is fixed at 0.80 (where $\mu = 1$ would be equivalent to standard logit). Along the x-axis of Figure 5, the membership of each good shifts between its retailer nest and its brand nest. At the x-axis value of -1, the demand structure is identical to that of a nested logit with nests based on *retailers* and nesting parameters $\mu = 0.80$. At the x-axis value of +1, the demand structure is identical to that of a nested logit with nests based on *brands* and nesting parameters $\mu = 0.80$. In between those values, each good has some degree of membership in each of its nests, and at $x = 0$ in the figure, each good has an equal degree of membership in both of its nests (yielding shares and substitution patterns similar to that of the standard logit model used in the baseline scenarios).

At the parameter values specified here, the greatest benefit from vertical integration occurs when nests are based on retailers, and the greatest harm from vertical integration occurs when nests are based on brands.

When nests are based on retailers, this means that a consumer inside of a retail location has a high propensity to make the purchase within that retailer, even if prices changes. In the extreme where the retailer nest is very important and the brand nest has no importance, retailers can be thought of as local monopolists.[15] In this case, the integrated brand cannot

[15] To have retailers as pure local monopolists, the model would also need that the nest parameter $\mu = 0$, while in this example μ has been set $\mu > 0$ so that retailers are highly differentiated but not quite local monopolists.

easily shift sales from downstream rivals to its integrated downstream partner. Thus, it does not have much incentive to raise rival retailers costs, and it does not gain leverage over those retailers by threatening to withhold its product: strong retailer nests means that if it stopped supply a rival retailer, consumers would mostly switch to other brands still sold in that retailer. With little scope for raising rival retailer costs, beneficial EDM effects can outweigh effects that harm consumers with linear pricing.

On the other hand, when brands are very important, there can be a large scope for raising rival retailer costs. When the brand nests are very strong, the integrated brand knows that it can easily shift sales to its brand at the integrated retailer. The integrated brand thus enjoys increased bargaining leverage over rival retailers and can negotiated higher wholesale prices. Substitution patterns where consumers are very willing to substitute across retailers involves greater scope to raise wholesale prices at rival retailers.

With two part tariffs, the patterns look similar at linear pricing, though with more scope for harm.

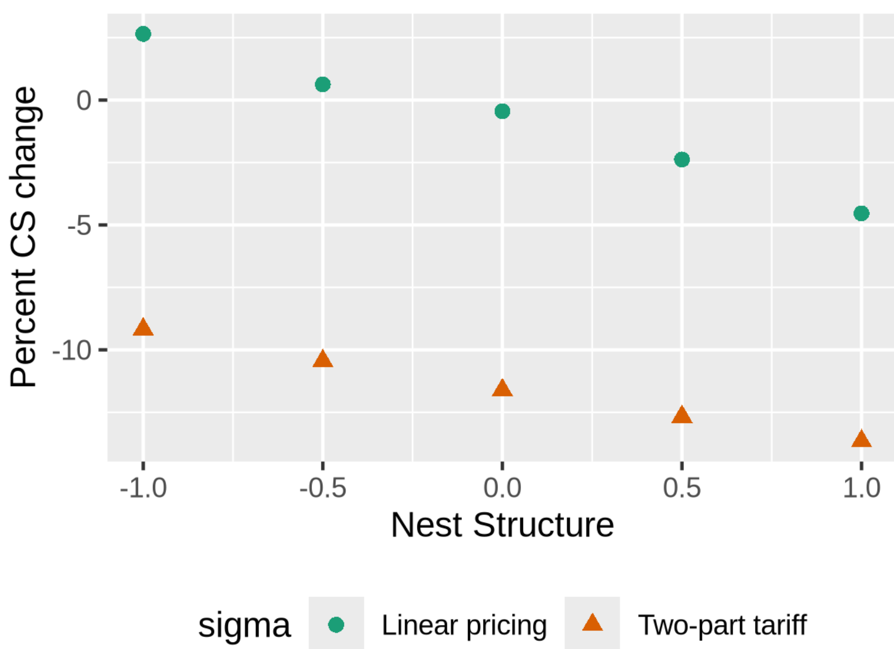

Figure 5: The x-axis value of -1 indicates a nested logit model with retailers as the nests. And value of +1 indicates a nested logit model with wholesaler brands as the nests. Intermediate values indicated a generalized nested logit model with products having membership in both retailer and brand nests.

5 Illustrative Example with Calibration

The previous section has looked at model properties holding the structural demand and costs parameters fixed, and only changing the strategic interactions. Practitioners of merger evaluation often face a different problem, which is to take observable market outcomes such as prices and quantities, and fit those to a model of competition. This is often done by using the model to calibrate demand and cost parameters that allow the model to fit the pre-merger observable prices and shares.

This section uses an empirical application to illustrate how calibration and simulation can be done with this model, and the consequences for predicted merger effects of the different models. Specifically, assume a symmetric market with four firms, and the practitioner observes: retail prices, wholesale prices, and market shares. In order to pose a just-identified model that avoids over-identification ambiguities, also suppose that the retail costs of the downstream merging partner are observed, and that the relative bargaining strengths of the upstream and downstream firms are known to be equal ($\lambda = 0.5$). Having a scenario with a just-identified model allows this example to focus on how model differences affect predictions about merger effects.[16]

This setup means that the demand parameters (α, δ), wholesale costs c^W, and retail costs c^R of the non-merging downstream firms need to be calibrated. The different models will imply different calibrated values of these parameters. Because this situation is just-identified, every model will imply exactly one set of parameters that will fit the model to the observed market outcomes.

The calibration approach takes four steps:

- Use the one observed downstream margin to calibrate the price coefficient $\hat{\alpha}$
- Use the calibrated value $\hat{\alpha}$ along with retail prices and market shares to find the mean utilities $\hat{\delta}$ that exactly match the model market shares to the observed market shares
- Use the calibrated demand parameters ($\hat{\alpha}$,$\hat{\delta}$) and retail prices p^R and market shares to

[16] Alternatively, one could assume that some wholesale costs were known, and the bargaining parameter could be calibrated. This approach has the intuitive appeal that costs are typically more readily observable than bargaining parameters. However, calibrating the bargaining parameter in the two-part tariff model requires observing the lump-sum payments. If lump-sum payments were observed, we would know that the linear price models are incorrect. In order to "just-identify" all of the models in as similar a way as possible, for the purposes of this exercise it is cleaner to assume the bargaining parameter is known and that it is unknown whether a lump-sum payment is made. All of the models then make predictions about upstream margins with the same observed information, allowing one to calibrate the wholesale costs. In practice, an analyst would more likely obtain wholesale cost information and be able to calibrate the bargaining parameter.

find the retail costs $\hat{c^R}$ that match the model's equilibrium retail prices to the observed retail prices

- Use the downstream model and the assumed value of the bargaining parameter λ to calibrate wholesale costs that allow the model to match its equilibrium wholesale prices to the observed wholesale prices the wholesale prices p^W

I focus on three models: (1) the sequential timing model with linear prices, (2) the sequential timing model with a full two-part tariff, and (3) the simultaneous timing model. For each model, I first calibrate the parameters that produce a symmetric equilibria with retail prices of 10, wholesale prices of 4, and market shares of 20% for the four goods in the model and a 20% share for an outside option. I then conduct a merger simulation exercise for each model.

Table 3 shows the comparisons across the three models. The first columns show the pre-merger fit for each model based on retail prices, wholesale prices, and market shares. Because the situation posited is just identified, each model is able to approximate the pre-merger outcomes.

The second set of columns of Table 3 show these values after a merger occurs between Retailer 1 and Wholesaler 1. Good 1 becomes an integrated good after this merger. The last two columns show the price change for each good, and the change in consumer surplus resulting from the merger.

As shown in the results, the different models make drastically different predictions about the effect on consumer welfare, even with the same observable market. Interestingly though, the relative changes in retail prices of the goods are the same. This indicates that each model is conveying similar effects of changes in economic incentives. Good 1, the good that becomes integrated, tends to experience a relative price decline and a relative increase in market share. Good 2, which is a rival brand offered inside of the integrated firm's retail partner, tends to experience a relative price increase and a decrease in share compared to pre-merger world. Good 3, which is the merging firm's brand offered inside of a rival retailer, experiences a relative increase in price and a decline in share compared to the pre-merger state of the world. Finally, good 4 is an independently owned brand inside of an independent retailer, and this good experiences a relative decline in price and increases in market share.

However, these relative price changes manifest through different overall effects. In the sequential model with linear pricing, Good 1's relative price declines due to an absolute decline in its retail prices. This leads to an overall increase in consumer surplus in the market. On the other hand, the sequential model with a two part tariff achieves the relative price decline

Table 3: Empirical Application

	Pre-Merger			Post-Merger			
Good	p_R	p_W	Share	p_R	p_W	Share	% Change p_R
Sequential: Linear prices							
1	10	4	20.3	9.2	-	30.5	-8.0
2	10	4	19.6	10.9	4.3	11.5	8.9
3	10	4	20.0	10.1	4.2	18.2	1.0
4	10	4	20.1	9.9	3.9	20.5	-1.1
Sequential: Two part tariff							
1	10	4	20.0	10.0	-	22.0	0.2
2	10	4	20.0	10.3	3.9	19.1	2.8
3	10	4	20.0	10.8	5.0	14.1	8.3
4	10	4	20.0	10.0	4.2	22.3	0.0
Simultaneous Model							
1	10	4	20.0	8.4	-	40.8	-15.9
2	10	4	20.0	11.1	3.7	9.2	11.0
3	10	4	20.0	11.1	5.4	9.2	11.0
4	10	4	20.0	9.4	3.7	23.9	-6.2

[a] Predicted merger effects from three different models in a symmetric market with four goods each with a retail price of 10, wholesale price of 4, market share of 20%, and an outside option with a 20% share. The percent change in consumer surplus is +2.2% in sequential linear pricing, -6.7% in sequential with two part tariff, and +10.5% in the simultaneous model.

of Good 1 by increasing the prices of the other goods in the market and keeping the price of Good 1 fairly similar post-merger. This yields net consumer harm resulting from the merger. In the simultaneous model, the absolute price decreases and price increases are both more pronounced. The overall effects balance out so that on net, this is a very beneficial merger.

This section has illustrated both how the models can all be readily implemented with the information typically available to practitioners of merger review, as well as how the assumed model of competition can yield drastically different vertical merger predictions for the same observed market prices and shares.

6 Discussion

This model accounts for many effects arising from vertical integration, yet many other potential effects are not accounted for. The following is a concise summary of the effects of vertical integration accounted for in this model:

- Downstream Effects
  - **EDM effect on integrated good.** This effect operates through the first-order condition for the integrated good of the retailer profit function (Equation 7). This effect puts downward pressure on the retail price of the integrated good. It is stronger depending on the amount of pre-merger double margin distortion and the own-price elasticity of the integrated good.
  - **Wholesale UPP effect on integrated good ("Chen effect").** This effect also operates through the first-order condition for the integrated good of the retailer profit function (Equation 7). This effect puts upward pressure on the retail price of the integrated good. Its strength depends on the size of the wholesale margins for brands sold by the upstream division of the integrated firm at rival retailers, and the diversion of sales from the integrated brand in the downstream division to the brand at rival retailers. Moresi and Salop 2021 call this the "Chen effect," after Chen 2001.
  - **Edgeworth-Salinger effect.** This effect operates through the first-order condition for non-integrated goods of the retailer profit function (Equation 6). This effect puts upward pricing pressure on rival brands sold through the integrated retailer. Its source is that integrated goods can become more profitable for the integrated retailer, and increasing the price of non-integrated goods can shift sales

to the integrated good. Thus the strength of this effect depends on the size of the EDM effect. Luco and Marshall 2020 call this the "Edgeworth-Salinger Effect."[17]

  - **Wholesale UPP effect on non-integrated goods.** This effect operates through the first-order condition for non-integrated goods of the retailer profit function (Equation 6). It puts upward pricing pressure on the retail price of rival brands sold inside the integrated retailer through recapture of sales by brands owned by the upstream division and sold through rival retailers. The strength of this effect depends on the size of the wholesale margins of brands owned by the upstream division when sold at rival retailers, and the strength of substitution from rival brands sold inside the integrated retailer to owned brands sold at rival retailers. In most cases this effect is likely to be small, but it is a theoretic possibility and a feature of demand systems such as standard logit.

- Upstream Effects
  - **Bargaining RRC effect.** This effect operates through the Nash Bargaining equation for negotiations between the integrated upstream division and rival downstream retailers. Because the integrated upstream division gains bargaining leverage vis a vis rival retailers t, this effect puts upward pricing pressure on the wholesale prices negotiated between the integrated brand and rival retailers, $p^W_{(t,w)}$. The strength of this effect depends, when in disagreement between the integrated brand w and a rival retailer t, how many sales are recaptured at the downstream retail division of the integrated firm. This includes both integrated products and non-integrated products at the downstream firm, and thus also depends on the amount of EDM on the integrated products and the size of the retail margins on the non-integrated products sold.
  - **Bargaining EDM effect.** This effect operates through the Nash Bargaining equation for negotiations between the integrated downstream division and rival upstream wholesalers. When the integrated retailer r ends in disagreement with a rival brand v, it now captures some of those sales at its integrated good. The sales at the integrated good are now more profitable if EDM has been realized, giving the integrated retailer more relative bargaining leverage. This puts downward pressure on the wholesale price of rival brands sold in the integrated retailer. The

[17] They are crediting F. Y. Edgeworth's work on how taxes affect the pricing incentives of multiproduct firms (Edgeworth 1925) and Michael Salinger's theoretical work linking Edgeworth's insights to the analysis of vertical mergers (Salinger 1991). It is also present in the Sheu and Taragin 2021 model, although they do not discuss it.

size of this effect depends on the amount of EDM realized on the integrated good and on the amount of diversion from rival brands to the jointly owned brand inside of the integrated retailer.

– **Bargaining recapture leverage effect.** Similar to the previous effect, this effect operates through the Nash Bargaining equation for negotiations between the integrated downstream division and rival upstream wholesalers. And similar to before this puts downward pressure on the wholesale price of rival brands sold in the integrated retailer. The difference is that this effect accounts for increased leverage to the downstream division through sales that, in disagreement with a rival brand, are recaptured through sales of jointly owned brands that are sold through other retailers. In most cases, this effect is likely to be small, but it is theoretically possible and a feature of some demand systems such as standard logit.

Previous literature has discussed how raising rival's cost incentive are intertwined with EDM (Das Varma and De Stefano 2020). Indeed, this is true for many of the effects accounted for in this model, such as the Edgeworth-Salinger effect and part of the Bargaining RRC effect, where the strength of these effects depend on the amount of EDM that is realized after integration. However, there are effects that put upward pressure on prices and that do not depend on EDM, such as the Wholesale UPP effects and the Bargaining recapture leverage effect. This is why in merger simulations with the two-part tariff there can still be meaningful consumer harm.

7 Conclusion

This paper presents a sequential model of a vertical supply chain with upstream bargaining and downstream price setting that can be readily used to evaluate vertical integration. The model importantly incorporates both linear and non-linear pricing, which is important because the amount of double marginalization drives predictions about vertical effects. I explore the properties of the model using numerical simulations, and show how the model's predictions compare to the same model with a simultaneous timing assumption that has been used in previous literature. I also illustrate how calibration and merger simulation can be implemented with this model using inputs that are typically observable to practitioners.

The previous section discussed the various changes in incentives that can be accounted for in this model. There are also several types of potential vertical effects that are not accounted for in this model. This study has ignored cost changes such as cost-saving efficiencies accruing to the integrating firms, or cost increases due to dis-economies of scale that rival firms may experience when they lose volume. A full analysis should account for any such changes in costs in order to evaluate the effects on consumer welfare of these other channels of benefits and harms.

Moreover, this model does not account for many types of effects of vertical integration including the effect on incentives to innovate, effects on barriers to entry or expansion, or how integration can change the likelihood of coordination in a market. Also, this model has assumed full supply networks, meaning that every brand is available in every retailer. Thus, this model is not appropriate to consider exclusive arrangements and how integration can change incentives for exclusive supply agreements. For situations where downstream firms use RFPs to procure a single input supplier, the sequential model of Podwol and Raskovich 2021 would be more appropriate. Their vertical model uses a second price auction upstream rather than Nash bargaining. More generally, incorporating a model of network formation such as those developed in Ho and Lee 2019, Ghili 2022, Liebman 2022, or Rey and Vergé 2020 would be a promising area of further work.

Sometimes, manufacturing firms have some degree of direct influence over retail prices, such as when they impose retail price maintenance (RPM) pricing policies. Such policies can have beneficial effects to discourage free-riding by downstream firms, or harmful effects such as by softening downstream price competition. Further research could look at extensions to this model to account for various pricing policies that afford upstream firms some direct control over downstream prices.

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A Two-Part Tariff

Two-Part Tariff with Sequential Timing

With sequential timing, it is straightforward to incorporate the two-part tariff. In the first stage of the game, agents negotiate over wholesale prices. In the second stage, downstream firms take the wholesale prices as given, and set retail prices to maximize profits. The firms can solve this game through backward induction, and thus the firms' negotiations upstream have direct influence on downstream outcomes.

In the sequential model with a two-part tariff, retailer r and wholesaler w negotiate over both p_j^W, the wholesale price of good j, and F_j, a lump-sum payment.

$$\max_{p_j^W, F_j} \left(\Pi^r - \widetilde{\Pi^r}(j) - F_j\right)^{\lambda} \left(\Pi^w - \widetilde{\Pi^w}(j) + F_j\right)^{1-\lambda} \qquad \forall j \in J$$

Maximizing with respect to F,

$$\begin{aligned}
0 &= \lambda \cdot \left(\Pi^r - \widetilde{\Pi^r}(j) - F_j\right)^{\lambda-1} \cdot (-1) \cdot \left(\Pi^w - \widetilde{\Pi^w}(j) + F_j\right)^{1-\lambda} \\
&\quad + \left(\Pi^r - \widetilde{\Pi^r}(j) - F_j\right)^{\lambda} \cdot (1-\lambda) \cdot \left(\Pi^w - \widetilde{\Pi^w}(j) + F_j\right)^{-\lambda} \\
&= \lambda \cdot (-1) \cdot \left(\Pi^w - \widetilde{\Pi^w}(j) + F_j\right) + \left(\Pi^r - \widetilde{\Pi^r}(j) - F_j\right)(1-\lambda) \\
\implies F_j &= (1-\lambda) \cdot (\Pi^r - \widetilde{\Pi^r}(j)) - \lambda \cdot (\Pi^w - \widetilde{\Pi^w}(j))
\end{aligned}$$

Plugging F back in to the Nash Product objective function yields an expression that is proportional to the joint surplus created:

$$
\begin{aligned}
&\max_{p_j^W} \left(\Pi^r - \widetilde{\Pi^r}(j) - F_j\right)^{\lambda} \left(\Pi^w - \widetilde{\Pi^w}(j) + F_j\right)^{1-\lambda} \\
\iff &\max_{p_j^W} \left(\Pi^r - \widetilde{\Pi^r}(j) - (1-\lambda)\cdot(\Pi^r - \widetilde{\Pi^r}(j)) + \lambda\cdot(\Pi^w - \widetilde{\Pi^w}(j))\right)^{\lambda} \\
&\qquad \left(\Pi^w - \widetilde{\Pi^w}(j) + (1-\lambda)\cdot(\Pi^r - \widetilde{\Pi^r}(j)) - \lambda\cdot(\Pi^w - \widetilde{\Pi^w}(j))\right)^{1-\lambda} \\
\iff &\max_{p_j^W} \left(\lambda\cdot(\Pi^r - \widetilde{\Pi^r}(j)) + \lambda\cdot(\Pi^w - \widetilde{\Pi^w}(j))\right)^{\lambda} \\
&\qquad \left((1-\lambda)\cdot(\Pi^r - \widetilde{\Pi^r}(j)) + (1-\lambda)\cdot(\Pi^w - \widetilde{\Pi^w}(j))\right)^{1-\lambda} \\
\iff &\max_{p_j^W} \lambda^{\lambda}\cdot(1-\lambda)^{1-\lambda}\cdot(\Pi^r - \widetilde{\Pi^r}(j) + \Pi^w - \widetilde{\Pi^w}(j))
\end{aligned}
$$

What this means is that the negotiating firms have an extra tool to use through the two-part tariff. In linear pricing, the wholesale price must be used to split the surplus. With the two-part tariff, the negotiating firms can now use the wholesale prices to maximize their joint profits, and then use the lump-sum payment F to split the surplus according to the bargaining weight λ. Put another way, the two-part tariff allows for efficiency gains because the wholesale price can be determined independently of the bargaining parameter.

The whole price p_j^W will satisfy the first-order condition in equilibrium. Recall that j denotes a product designating brand w available in retailer r, and that W^r denotes the set of brands available in retailer r, and R^w denotes the set of retailers in which brand w is available. Then the first order condition of the Nash Product objective function with respect to $j = (r, w)$ is:

$$
\frac{\partial NP}{\partial p_j^W} = L \cdot \left. \sum_{x \in W^r} (p_{rx}^R - p_{rx}^W - c_{rx}^R) \cdot \frac{\partial s_{rx}}{\partial p_{rw}^W} + \sum_{t \in R^w} (p_{tw}^W - c_{tw}^W) \cdot \frac{\partial s_{tw}}{\partial p_{rw}^W} \right) = 0 \tag{8}
$$

where the constant $L = \lambda^{\lambda} \cdot (1-\lambda)^{1-\lambda}$.

In the sequential model, $\frac{\partial s_{rx}}{\partial p_{rw}^W} \neq 0$ and $\frac{\partial s_{tw}}{\partial p_{rw}^W} \neq 0$, and there is a set of wholesale prices that will satisfy the bargaining FOCs.

Two-Part Tariff with Simultaneous Timing

In contrast to sequential timing, the two-part tariff is not as straightforward to incorporate into a model with simultaneous timing.

With the simultaneous timing, upstream negotiations occur holding the downstream fixed, and downstream price setting occurs holding the upstream negotiations fixed. This means that the upstream negotiations have no direct way to influence the downstream outcomes, and so the wholesale prices cannot be used to directly induce efficient outcomes in the downstream market. Formally, this is because the market shares are assumed based on the timing assumption to have independence from the wholesale prices: that is, $\frac{\partial s_j}{\partial p_j^W} = 0$. This condition is required to hold in order to derive a closed form solution for the first-order conditions from the Nash Product (Equation 4 in Sheu and Taragin 2021, or Equation 6 in Ho and Lee 2017). Another consequence of this, however, is that the upstream negotiations cannot be used by the firms to directly affect downstream outcomes.

As long as $\frac{\partial s_k}{\partial p_j^W} = 0$, the first order condition in Equation 8 is satisfied for any value of p_j^W, and the Nash Product $\max_{p_j^W}(\Pi^r + \Pi^w)$ is maximized. This is because with the simultaneous timing and the corresponding condition, p_j^W is not changing the total amount of surplus available. And because the lump sum payment allows for the Nash Product to be independent of the bargaining parameter, there is nothing to pin down unique wholesale prices. Any values of p_j^W can satisfy an equilibrium.

Additional information or structure could be imposed to pin down wholesale prices. For example, one could assume that wholesale prices are set efficiently at $p_j^W = c_j^W$, and the surplus is then split through a lump sum payment according to the bargaining parameter. An equilibrium set of retail prices, wholesale prices, and lump sum payments could then be computed. But unique wholesale prices do not arise endogenously as a result of the bargaining problem in this model. Consequently, I do not consider a two-part tariff in the simultaneous model in this paper.

B Appendix Tables

Table 4: Baseline details

		Pre-Merger			Post-Merger				
Sigma	Good	p_R	p_W	Share	p_R	p_W	Share	% Change p_R	% Change CS
0.0	1	2.5	0.9	21.7	2.2	-	32.3	-13.6	0.6
0.0	2	2.5	0.9	21.5	3.0	1.0	12.4	17.7	0.6
0.0	3	2.5	0.9	21.6	2.6	1.0	19.2	3.5	0.6
0.0	4	2.5	0.9	21.7	2.5	0.8	22.8	-2.0	0.6
0.2	1	2.5	0.8	21.7	2.2	-	31.8	-12.4	-0.5
0.2	2	2.5	0.8	21.7	2.9	0.9	13.2	17.0	-0.5
0.2	3	2.5	0.8	21.7	2.6	1.0	18.8	5.1	-0.5
0.2	4	2.5	0.8	21.8	2.5	0.8	23.0	-1.5	-0.5
0.4	1	2.5	0.8	22.0	2.2	-	31.4	-10.6	-2.1
0.4	2	2.5	0.8	21.7	2.9	0.9	13.8	16.7	-2.1
0.4	3	2.5	0.8	22.0	2.6	1.0	18.2	7.9	-2.1
0.4	4	2.5	0.8	21.8	2.4	0.8	23.4	-1.0	-2.1
0.6	1	2.4	0.7	21.9	2.2	-	30.6	-8.8	-3.8
0.6	2	2.4	0.7	22.1	2.8	0.8	15.0	16.1	-3.8
0.6	3	2.4	0.7	21.9	2.7	1.1	17.4	10.7	-3.8
0.6	4	2.4	0.7	22.1	2.4	0.8	23.9	0.0	-3.8
0.8	1	2.3	0.6	22.2	2.2	-	29.6	-5.1	-6.7
0.8	2	2.3	0.6	22.3	2.7	0.7	16.6	15.7	-6.7
0.8	3	2.3	0.6	22.2	2.7	1.1	16.2	16.4	-6.7
0.8	4	2.3	0.7	22.1	2.4	0.8	24.6	1.4	-6.7
1.0	1	2.2	0.5	22.4	2.3	-	27.1	2.5	-11.1
1.0	2	2.2	0.5	22.7	2.5	0.4	20.6	13.3	-11.1
1.0	3	2.2	0.5	22.8	2.8	1.2	14.1	28.1	-11.1
1.0	4	2.2	0.5	22.5	2.3	0.7	25.6	4.7	-11.1

[a] Values underlying simulations shown in Figure 1.

[b] $\sigma = 0$ corresponds to linear pricing and $\sigma = 1$ corresponds to a two part tariff.

Table 5: Outside share effect details

		Pre-Merger			Post-Merger				
Sigma	Good	p_R	p_W	Share	p_R	p_W	Share	% Change p_R	% Change CS
0	1	2.3	0.8	17.6	1.9	-	26.7	-17.3	4.9
0	2	2.3	0.8	17.7	2.6	0.9	11.1	14.8	4.9
0	3	2.3	0.8	17.7	2.3	0.8	17.3	-1.4	4.9
0	4	2.3	0.8	17.7	2.3	0.8	17.3	-1.5	4.9
0	1	2.5	0.8	20.8	2.1	-	31.0	-14.5	1.6
0	2	2.5	0.8	20.6	2.9	1.0	12.2	16.8	1.6
0	3	2.5	0.8	20.6	2.5	0.9	18.7	2.3	1.6
0	4	2.5	0.8	20.8	2.4	0.8	21.5	-2.0	1.6
0	1	2.6	0.9	22.7	2.3	-	34.1	-12.5	-0.8
0	2	2.6	0.9	22.8	3.1	1.0	12.6	19.6	-0.8
0	3	2.6	0.9	22.8	2.8	1.1	19.1	6.3	-0.8
0	4	2.6	0.9	22.7	2.5	0.8	25.0	-2.4	-0.8
0	1	2.7	0.9	23.8	2.4	-	36.0	-10.9	-1.9
0	2	2.7	0.9	24.1	3.2	1.0	13.0	21.0	-1.9
0	3	2.7	0.9	24.0	2.9	1.2	19.1	9.0	-1.9
0	4	2.7	0.9	23.9	2.6	0.9	27.5	-2.4	-1.9
0	1	2.7	0.9	24.3	2.4	-	36.5	-10.6	-1.7
0	2	2.7	0.9	24.3	3.3	1.1	13.1	20.9	-1.7
0	3	2.7	0.9	24.2	3.0	1.2	18.7	9.8	-1.7
0	4	2.7	0.9	24.4	2.6	0.8	28.7	-3.1	-1.7
1	1	1.7	0.3	17.3	1.8	-	18.1	1.8	-7.3
1	2	1.8	0.3	17.0	1.8	0.3	16.8	4.5	-7.3
1	3	1.7	0.3	17.3	2.2	0.8	11.3	24.3	-7.3
1	4	1.8	0.3	17.0	1.7	0.3	19.6	-2.6	-7.3
1	1	2.0	0.4	20.4	2.0	-	22.9	1.5	-9.0
1	2	2.0	0.4	20.4	2.2	0.3	19.4	8.5	-9.0
1	3	2.0	0.4	20.5	2.5	1.0	12.8	26.1	-9.0
1	4	2.0	0.4	20.4	2.0	0.5	23.6	0.3	-9.0
1	1	2.2	0.5	22.4	2.3	-	27.1	2.5	-11.1
1	2	2.2	0.5	22.7	2.5	0.4	20.6	13.3	-11.1
1	3	2.2	0.5	22.8	2.8	1.2	14.1	28.1	-11.1
1	4	2.2	0.5	22.5	2.3	0.7	25.6	4.7	-11.1
1	1	2.4	0.6	23.9	2.5	-	30.5	5.7	-13.4
1	2	2.4	0.6	23.7	2.8	0.5	21.0	18.4	-13.4
1	3	2.4	0.6	24.0	3.1	1.5	15.8	29.0	-13.4
1	4	2.4	0.6	23.7	2.7	1.1	25.6	11.5	-13.4
1	1	2.5	0.7	24.7	2.8	-	33.0	10.7	-15.7
1	2	2.5	0.7	24.3	3.1	0.6	21.2	24.8	-15.7
1	3	2.5	0.7	24.5	3.3	1.7	17.2	32.0	-15.7
1	4	2.5	0.7	24.4	3.0	1.4	24.8	19.8	-15.7

[a] Values underlying simulations shown in Figure 2.
[b] $\sigma = 0$ corresponds to linear pricing and $\sigma = 1$ corresponds to a two part tariff.

Table 6: Pre-Merger integrated share details

		Pre-Merger			Post-Merger				
Sigma	Good	p_R	p_W	Share	p_R	p_W	Share	% Change p_R	% Change CS
0	1	2.2	0.7	5.7	1.8	-	9.2	-17.2	-1.4
0	2	2.5	1.1	28.4	2.7	1.1	22.2	8.9	-1.4
0	3	2.6	0.8	25.2	2.6	0.7	27.2	-1.7	-1.4
0	4	2.7	0.9	22.9	2.7	0.8	23.2	0.4	-1.4
0	1	2.3	0.8	12.0	2.0	-	18.7	-15.4	-0.7
0	2	2.5	1.0	26.0	2.8	1.1	18.3	12.1	-0.7
0	3	2.6	0.8	23.8	2.6	0.8	23.8	0.4	-0.7
0	4	2.6	0.9	22.4	2.6	0.8	23.2	-0.7	-0.7
0	1	2.5	0.9	21.7	2.2	-	32.4	-13.5	0.4
0	2	2.5	0.9	21.5	3.0	1.0	12.4	18.0	0.4
0	3	2.5	0.9	21.5	2.6	1.0	19.1	3.7	0.4
0	4	2.5	0.8	21.7	2.5	0.8	22.8	-1.9	0.4
0	1	2.8	1.0	33.4	2.5	-	47.3	-10.9	1.1
0	2	2.6	0.8	16.0	3.3	1.0	6.8	26.3	1.1
0	3	2.5	0.9	19.1	2.7	1.3	13.4	11.2	1.1
0	4	2.4	0.8	20.4	2.3	0.9	21.6	-2.8	1.1
0	1	3.3	1.2	44.7	3.0	-	60.5	-7.9	0.3
0	2	2.8	0.7	10.7	3.8	1.0	3.0	37.4	0.3
0	3	2.4	1.0	16.3	3.0	1.7	7.9	24.5	0.3
0	4	2.3	0.8	19.1	2.3	0.9	19.4	-0.9	0.3
1	1	1.9	0.4	5.6	2.0	-	6.5	2.0	-9.8
1	2	2.2	0.7	30.3	2.3	0.6	30.9	7.0	-9.8
1	3	2.3	0.4	26.7	2.8	1.0	18.1	21.6	-9.8
1	4	2.3	0.4	25.0	2.4	0.6	29.1	1.8	-9.8
1	1	2.0	0.4	12.1	2.1	-	14.3	2.2	-10.2
1	2	2.2	0.6	27.6	2.4	0.5	27.1	9.1	-10.2
1	3	2.3	0.4	25.0	2.8	1.1	16.8	23.0	-10.2
1	4	2.3	0.5	24.0	2.4	0.7	27.7	2.9	-10.2
1	1	2.2	0.5	22.5	2.3	-	27.1	2.5	-11.0
1	2	2.2	0.5	22.6	2.5	0.4	20.6	13.1	-11.0
1	3	2.2	0.5	22.6	2.8	1.2	14.2	27.0	-11.0
1	4	2.2	0.5	22.7	2.3	0.7	25.5	5.2	-11.0
1	1	2.5	0.6	36.2	2.6	-	42.6	4.0	-11.3
1	2	2.4	0.4	16.0	2.7	0.3	13.4	16.3	-11.3
1	3	2.2	0.6	19.3	2.9	1.5	10.4	34.0	-11.3
1	4	2.1	0.6	20.4	2.3	0.9	22.7	6.9	-11.3
1	1	2.9	0.7	50.0	3.1	-	57.7	4.9	-11.7
1	2	2.6	0.3	9.8	3.2	0.3	6.9	21.3	-11.7
1	3	2.2	0.8	15.8	3.2	1.9	6.6	44.8	-11.7
1	4	2.1	0.7	17.6	2.3	1.0	19.5	8.2	-11.7

[a] Values underlying simulations shown in Figure 3.
[b] $\sigma = 0$ corresponds to linear pricing and $\sigma = 1$ corresponds to a two part tariff.

Table 7: Effect of changing bargaining power

	Consumer Surplus			
Lambda	Pre	Post	Model	Percent Change CS
0.3	1.53	1.61	sequential linear pricing	5.0
0.4	1.57	1.63	sequential linear pricing	3.5
0.5	1.67	1.68	sequential linear pricing	0.3
0.6	1.78	1.78	sequential linear pricing	0.1
0.7	1.88	1.90	sequential linear pricing	0.7
0.3	1.94	1.77	sequential two part tariff	-8.7
0.4	1.94	1.77	sequential two part tariff	-8.7
0.5	1.94	1.77	sequential two part tariff	-8.7
0.6	1.94	1.77	sequential two part tariff	-8.7
0.7	1.94	1.77	sequential two part tariff	-8.7
0.3	0.57	1.00	simultaneous	75.7
0.4	0.90	1.17	simultaneous	29.9
0.5	1.20	1.33	simultaneous	11.1
0.6	1.46	1.48	simultaneous	1.2
0.7	1.69	1.61	simultaneous	-4.5

[a] Values underlying simulations shown in Figure 4.
[b] λ is the retailer relative bargaining power.

Table 8: Effect of changing bargaining power: product-level

		Pre-Merger			Post-Merger				
Model	Lambda	Good	p_R	p_W	Share	p_R	p_W	Share	% Change p_R
sequential linear pricing	0.3	1	2.7	1.0	21.0	2.2	-	33.2	-18.5
sequential linear pricing	0.3	2	2.7	1.0	21.0	3.1	1.0	12.0	14.8
sequential linear pricing	0.3	3	2.7	1.0	21.0	2.6	1.0	20.2	-3.7
sequential linear pricing	0.3	4	2.7	1.0	21.0	2.6	1.0	20.2	-3.7
sequential linear pricing	0.4	1	2.6	1.0	21.2	2.2	-	33.0	-15.4
sequential linear pricing	0.4	2	2.6	1.0	21.3	3.1	1.0	12.0	19.2
sequential linear pricing	0.4	3	2.6	1.0	21.2	2.7	1.0	19.3	3.8
sequential linear pricing	0.4	4	2.6	1.0	21.2	2.6	1.0	21.5	0.0
sequential linear pricing	0.5	1	2.5	0.8	21.7	2.2	-	32.3	-12.0
sequential linear pricing	0.5	2	2.5	0.9	21.6	3.0	1.0	12.5	20.0
sequential linear pricing	0.5	3	2.5	0.8	21.7	2.6	1.0	19.1	4.0
sequential linear pricing	0.5	4	2.5	0.8	21.6	2.5	0.8	22.8	0.0
sequential linear pricing	0.6	1	2.4	0.7	22.0	2.1	-	30.7	-12.5
sequential linear pricing	0.6	2	2.4	0.7	22.1	2.8	0.9	14.0	16.7
sequential linear pricing	0.6	3	2.4	0.7	22.0	2.5	0.8	20.6	4.2
sequential linear pricing	0.6	4	2.4	0.7	22.1	2.4	0.7	22.8	0.0
sequential linear pricing	0.7	1	2.3	0.6	22.5	2.1	-	28.7	-8.7
sequential linear pricing	0.7	2	2.3	0.6	22.3	2.5	0.7	16.3	8.7
sequential linear pricing	0.7	3	2.3	0.6	22.3	2.3	0.6	22.4	0.0
sequential linear pricing	0.7	4	2.3	0.6	22.4	2.3	0.6	22.4	0.0
simultaneous	0.3	1	4.1	2.8	12.4	2.5	-	50.5	-39.0
simultaneous	0.3	2	4.1	2.8	12.4	4.8	2.5	3.3	17.1
simultaneous	0.3	3	4.1	2.8	12.4	4.6	3.4	4.1	12.2
simultaneous	0.3	4	4.1	2.8	12.4	3.7	2.5	12.0	-9.8
simultaneous	0.4	1	3.5	2.1	16.5	2.4	-	44.2	-31.4
simultaneous	0.4	2	3.5	2.1	16.5	4.1	1.8	6.1	17.1
simultaneous	0.4	3	3.5	2.1	16.5	4.0	2.6	7.0	14.3
simultaneous	0.4	4	3.5	2.1	16.5	3.2	1.9	18.2	-8.6
simultaneous	0.5	1	3.1	1.5	19.1	2.4	-	39.3	-22.6
simultaneous	0.5	2	3.1	1.5	19.1	3.6	1.4	9.1	16.1
simultaneous	0.5	3	3.1	1.5	19.1	3.6	2.2	9.1	16.1
simultaneous	0.5	4	3.1	1.5	19.1	2.8	1.4	22.3	-9.7
simultaneous	0.6	1	2.8	1.1	20.7	2.3	-	35.2	-17.9
simultaneous	0.6	2	2.8	1.1	20.7	3.2	1.1	12.2	14.3
simultaneous	0.6	3	2.8	1.1	20.7	3.3	1.8	10.4	17.9
simultaneous	0.6	4	2.8	1.1	20.7	2.6	1.1	25.3	-7.1
simultaneous	0.7	1	2.5	0.8	21.7	2.3	-	31.8	-8.0
simultaneous	0.7	2	2.5	0.8	21.7	2.9	0.8	15.1	16.0
simultaneous	0.7	3	2.5	0.8	21.7	3.1	1.6	11.1	24.0
simultaneous	0.7	4	2.5	0.8	21.7	2.4	0.8	27.5	-4.0

[a] Values underlying simulations shown in Figure 4.

[b] λ is the retailer relative bargaining power.