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Merger Analysis with Latent Price

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Merger Analysis with Latent Price*

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Abstract

Standard empirical tools for merger analysis assume price data, which are often unavailable. I characterize sufficient conditions for identifying the unilateral effects of mergers without price data using the first-order approach and merger simulation. Data on merging firms' revenues, margins, and revenue diversion ratios are sufficient to identify their gross upward pricing pressure indices and compensating marginal cost reductions. Standard discrete-continuous demand assumptions facilitate the identification of revenue diversion ratios as well as the feasibility of merger simulation in terms of percentage change in price. I apply the framework to the Albertsons/Safeway (2015) and Staples/Office Depot (2016) mergers.

Keywords: Merger, unilateral effect, diversion ratio, upward pricing pressure, merger simulation

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1 Introduction

1.1 Motivation

A horizontal merger is said to generate unilateral effects when it reduces competition between the merging parties and enables them to increase prices. Standard empirical tools for evaluating mergers typically assume that price (and quantity) data are available (Davis and Garcés, 2009; Valletti and Zenger, 2021; Miller and Sheu, 2021). Merger simulations use price data to estimate or calibrate demand functions. The first-order approach that measures unilateral effects via upward pricing pressures and compensating marginal cost reductions requires price data as either direct or indirect inputs to the formulas and for estimating demand slopes and curvatures.

However, researchers often have difficulty accessing reliable price and quantity data. First, price and quantity data may not exist. Second, even if the data exists, the researcher may not have access. Third, even if the researcher can access the data, it may be costly to process it. For example, if a merger involves retail firms (e.g., supermarket chains) whose outlets carry thousands of non-overlapping items with prices that frequently vary due to complex promotion activities, constructing relevant price indices can be non-trivial and time-consuming. The lack of "clean" price data often creates considerable challenges for economists and antitrust agencies in predicting the price and welfare effects of mergers.

1.2 Main Findings

This article develops an empirical framework for estimating the unilateral effects of horizontal mergers without price data, extending the scope of applicability of the first-order approach and merger simulation. I consider the standard empirical Bertrand-Nash multiproduct oligopoly framework. Unlike the standard merger analysis framework, however, I study the identifiability of unilateral effects based on revenue and margin data while treating prices as "latent" (i.e., unobserved). My framework bypasses the standard data assumption that prices and quantities are observed separately. My key idea is to focus on *revenue diversion ratios* instead of *quantity diversion ratios*. Whereas quantity diversion ratios have served as primary measures of demand-side substitutability between products since Shapiro (1996), revenue diversion ratios have received little attention.

I show that, given the assumptions of my framework, data on merging parties' revenues, margins, and revenue diversion ratios are sufficient to identify their gross upward pricing pressure indices and compensating marginal cost reductions. Using the definition of revenue diversion ratios, I can express firms' first-order conditions and gross upward pricing pressure statistics as functions of their margins, revenue diversion ratios, and own-price demand elasticities. In turn, the own-price demand elasticities of products can be identified from the owner firm's margins and revenue diversion ratios data via the profit maximization condition. Thus, the gross upward pricing pressure statistics are identifiable from the merging firms' revenues, margins, and revenue diversion ratios. Analogous arguments show that the same data assumption identifies compensating marginal cost reductions.

If estimates of merger pass-through rates are available, the analyst can translate gross upward pricing pressure indices to first-order merger price and welfare effects (Jaffe and Weyl, 2013). Although estimating pass-through rates in imperfect competition settings can be challenging, I show that when consumers have CES preferences, the merger pass-through matrix can also be calculated from merging firms' revenue and margin data. When estimates of merger pass-through rates are unavailable, the analyst can apply Miller et al. (2017b)'s approach, which says that replacing the unknown merger pass-through matrix with an identity matrix can reasonably approximate the true merger price effects.

I show the standard discrete-continuous demand assumption facilitates the identification of revenue diversion ratios and merger simulation. Unlike pure discrete-choice demand models in which consumers make a discrete choice with unit-inelastic quantity demanded, I assume consumers make a discrete choice for every unit of their budget. The expenditure shares are then derived with the standard additive random utility assumption, which allows me to leverage standard tools from the GEV discrete choice literature. If the analyst knows the distribution of random utility shocks and consumers have homogeneous responsiveness to price, consumers' expenditure data (or product-level revenues if there is a single representative consumer) are sufficient to identify revenue diversion ratios via the Hotz and Miller (1993)'s inversion theorem.[1] In particular, assuming consumers have CES preferences substantially simplifies the econometric problem since their expenditure shares admit multinomial logit probability (softmax) forms, and the revenue diversion ratios can be identified from consumers' expenditures on merging firms' products or second-choice data.

[1]Homogeneous price responsiveness assumption, although relatively standard, may be strong as it rules out more flexible demand systems such as random coefficient models. However, relaxing the assumption without price data appears non-trivial and is beyond the scope of this article.

Next, I turn to merger simulation and provide sufficient conditions for running merger simulations without price data, complementing the merger simulation literature. I show that although the post-merger price levels cannot be identified, the percentage change in price in the post-merger equilibrium relative to pre-merger levels can be identified. The key step is to express post-merger optimal pricing conditions as known functions of percentage deviation in prices from the per-merger values, which I achieve with the log-linearity in price assumption. Expressing post-merger first-order conditions as known functions of percentage deviation in price requires the analyst to observe all competitors' revenues and margins, so merger simulation is more demanding regarding data requirements than the first-order approach. In practice, the analyst can estimate non-merging firms' margins by leveraging demand assumptions with information on merging firms' margins.

I illustrate my methodology's usefulness using two empirical applications. First, I apply the framework to the Staples/Office Depot merger, which was proposed in 2014 but eventually blocked in 2016. This empirical example is chosen to illustrate the simplicity of my approach. In its complaint, the FTC claimed a cluster market for consumable office supplies that includes many individual items. The nature of the claimed market makes it challenging to construct price data that fit standard econometric frameworks. My framework does not require price data. Using publicly available data on the firms' revenues, margins, and market shares, I evaluate the merger's unilateral effects using the first-order approach and merger simulation. All approaches predict either the merger produces substantial harm or requires significant cost reductions to offset upward pricing pressures.

Next, I apply the framework to analyze the Albertsons/Safeway merger, the largest grocery merger in US history. This empirical example illustrates the scalability of my approach. The Federal Trade Commission approved the merger in 2015, conditional on divesting 168 stores. Using Nielsen's Trade Dimensions data, I estimate a spatially aggregated nested CES demand model using cross-sectional data on store revenues to estimate revenue diversion ratios. I then estimate store-level merger price/welfare effects for thousands of stores before and after the divestiture.[2] My estimates suggest that the FTC-mandated divestiture significantly reduced annual consumer harm. Finally, I estimate the distribution of cost efficiencies that would be required for my model

[2]I use my framework to predict merger effects using pre-merger data. I do not conduct a merger retrospective, which would use pre- and post-merger data to estimate the actual effects of a merger.

to predict zero post-merger price effect (i.e., offset upward pricing pressure).

1.3 Relationship to the Literature

This article mainly contributes to the merger analysis literature by developing an empirical framework for analyzing horizontal mergers without price data. I highlight four contributions relative to the previous literature. First, this article complements the literature that develops empirical tools for analyzing mergers with easy-to-calculate sufficient statistics (Werden, 1996; Farrell and Shapiro, 2010; Jaffe and Weyl, 2013; Affeldt et al., 2013; Miller et al., 2013; Weyl and Fabinger, 2013; Brito et al., 2018; Miller and Sheu, 2021). I provide a novel identification argument for identifying first-order unilateral effect statistics based on revenue-based measures of diversion ratios.[3] My identification arguments relax the data requirements in Werden (1996), Farrell and Shapiro (2010) and Jaffe and Weyl (2013), who assumed prices and quantities are observed separately (along with margins).[4]

Second, this article contributes to a large body of research on merger simulation (Hausman et al., 1994; Werden and Froeb, 1994; Nevo, 2000; Epstein and Rubinfeld, 2001; Werden and Froeb, 2002). I show that I can calculate percentage changes in price without estimating a complete set of demand function parameters. My results are similar to Miller (2014, 2017)'s result that shows percentage changes in markups can be identified from merging firms' market shares in procurement settings.

Third, this article contributes to a body of research that finds the value of employing discrete-continuous demand assumptions for theoretical and empirical merger analysis (Anderson et al., 1987, 1988, 1992; Björnerstedt and Verboven, 2016; Nocke and Schutz, 2018; Nocke and Whinston, 2022; Taragin and Sandfort, 2022; Caradonna et al., 2023; Nocke and Schutz, 2023; Garrido, 2024). Employing a discrete-continuous demand assumption facilitates the identification of revenue diversion ratios and merger simulation when price data are absent. Specifically, assuming consumers' expenditures to products are generated from an additive random utility model for every unit of

[3] Revenue diversion ratios as measures of consumer substitution have received little attention in the literature. The only exception I know of is Caradonna et al. (2023), which studies merger-induced entries with (nested) multinomial and CES demand systems.

[4] There may be circumstances where the standard approach is preferred over my approach in the absence of price data. For example, if revenues are unobserved but some measures of quantity sold are observed (e.g., the number of customer visits to stores or hospitals), the analyst may estimate quantity diversion ratios assuming consumers have logit demand (Ferguson et al., 2023).

budget allows the analyst to apply the standard econometric results from the GEV discrete choice literature. While a common approach to calculating gross upward pricing pressure indices in the absence of price data has been to use revenues as proxies for quantities or assume prices of merging firms' products are approximately equal (Ferguson et al., 2023), I show that such ad hoc assumptions are unnecessary. This article also relates to a body of research that recognizes how CES demand assumption allows the analyst to estimate production functions with revenue data when prices and quantities are unobserved (Klette and Griliches, 1996; De Loecker, 2011; Grieco et al., 2016; Gandhi et al., 2020).

Finally, my empirical applications complement the existing body of research on retail merger retrospectives and divestiture remedies (Smith, 2004; Hosken et al., 2016; Allain et al., 2017; Thomassen et al., 2017; Hosken et al., 2018; Ellickson et al., 2020). My empirical applications are new in the literature. The closest to my work are Smith (2004) and Ellickson et al. (2020), which also study supermarket mergers. Smith (2004) also uses profit margins data and equilibrium pricing conditions to analyze supermarket competition but develops a structural model tailored to fit consumer shopping patterns data in the supermarket industry. In contrast, my framework follows the standard empirical Bertrand Nash pricing model. Ellickson et al. (2020) develops a spatial demand framework that overcomes the absence of price data in the grocery competition setting but does not directly calculate merger price effects. My approach can calculate merger price effects under the same set of assumptions.

1.4 Outline

The rest of the article is organized as follows. In Section 2, I describe the firm-side model and introduce the concept of revenue diversion ratios. In Section 3, I establish identification conditions for gross upward pricing pressure indices and compensating marginal cost reductions. In Section 4, I characterize consumer demand assumptions that facilitate the estimation of revenue diversion ratios. In Section 5, I show merger simulation is feasible. In Sections 6 and 7, I apply the proposed methodology to evaluate the Staples/Office Depot merger (2016) and the Albertsons/Safeway merger (2015). Finally, I conclude in Section 8. All proofs are in Appendix A. Online Appendix can be found at the author's website.

2 Model

In this section, I describe the firm-side model. I also introduce the concept of revenue diversion ratio and discuss its relationship with quantity diversion ratios.

2.1 Setup

The firm-side model primitives are summarized by a tuple $\langle \mathcal{J}, \mathcal{F}, (\pi_j)_{j\in\mathcal{J}} \rangle$, where $\mathcal{J}$ is the set of products, $\mathcal{F}$ is the set of multiproduct firms, and π_j specifies the product j's profit function. Set $\mathcal{F}$ forms a partition over $\mathcal{J}$, specifying firms' ownership over products. The set of products owned by firm $F \in \mathcal{F}$ is denoted $\mathcal{J}_F \subseteq \mathcal{J}$. I assume the profit from product $j \in \mathcal{J}$ is $\pi_j = (p_j - c_j)q_j$ where $p_j \in \mathbb{R}_+$, $c_j \in \mathbb{R}_+$, and $q_j = q_j(p)$ denote price, constant marginal cost, and quantity demanded, respectively; I omit fixed costs for notational convenience. I also assume that the products are substitutes. I use $m_j \equiv (p_j - c_j)/p_j$ to denote relative margins.

2.2 Firm's Problem

The multiproduct firms engage in a Bertrand-Nash pricing game. Each firm $F \in \mathcal{F}$ maximizes its total profit $\sum_{j\in\mathcal{J}_F} \pi_j$ with respect to a vector of prices $(p_j)_{j\in\mathcal{J}_F}$. Normalizing the first-order conditions to be quasilinear in margins yields

$$-\epsilon_{jj}^{-1} - m_j + \sum_{l\in\mathcal{J}_F\setminus j} m_l D_{j\to l}\frac{p_l}{p_j} = 0, \tag{1}$$

where $\epsilon_{jj} \equiv \frac{\partial q_j}{\partial p_j}\frac{p_j}{q_j}$ is the own-price elasticity of demand, and

$$D_{j\to l} \equiv -\frac{\partial q_l/\partial p_j}{\partial q_j/\partial p_j} \tag{2}$$

is the quantity diversion ratio from product j to product l. I defer derivations of firms' optimal pricing equations, gross upward pricing pressure indices, and compensating marginal cost reductions to Online Appendix A as they are standard.

2.3 Revenue Diversion Ratios and Their Properties

Definition

Standard empirical frameworks for mergers' unilateral effects analysis have focused on *quantity diversion ratios* (2) as the key statistics for measuring substitutability between products (Shapiro, 1996; Farrell and Shapiro, 2010; Conlon and Mortimer, 2021). To establish the identifiability of merger price effects without price data, I will rewrite price elasticities and gross upward pricing pressure indices in terms of *revenue diversion ratios.* Revenue diversion ratio from product j to k is defined as

$$D^R_{j\to k} \equiv -\frac{\partial R_k/\partial p_j}{\partial R_j/\partial p_j}, \tag{3}$$

where $R_l \equiv p_l * q_l$ is product l's revenue. It measures the substitutability between two products by studying how *revenue* shifts from one product to another following a unilateral price increase. Under standard regularity conditions, revenue diversion ratios are non-negative in equilibrium.[5]

Relationship to Quantity Diversion Ratios

Revenue and quantity diversion ratios are different, but they are closely related. Let $\epsilon_{jj} = \frac{\partial q_j}{\partial p_j}\frac{p_j}{q_j}$ and $\epsilon^R_{jj} = \frac{\partial R_j}{\partial p_j}\frac{p_j}{R_j}$ denote the own-price elasticity of demand and the own-price elasticity of revenue, respectively. Similarly, let $\epsilon_{kj} = \frac{\partial q_k}{\partial p_j}\frac{p_j}{q_k}$ and $\epsilon^R_{kj} = \frac{\partial R_k}{\partial p_j}\frac{p_j}{R_k}$ denote the cross-price elasticities. The following lemma summarizes their relationship.

Lemma 1 (Relationship between revenue-based and quantity-based measures)**.** *For an arbitrary pair of products j and k,*

1. $D^R_{j\to k} = -\frac{\epsilon^R_{kj}}{\epsilon^R_{jj}}\frac{R_k}{R_j}$ *and* $D_{j\to k} = -\frac{\epsilon_{kj}}{\epsilon_{jj}}\frac{q_k}{q_j}$*;*

2. $\epsilon^R_{jj} = \epsilon_{jj} + 1$*, and* $\epsilon^R_{kj} = \epsilon_{kj}$ *for* $j \neq k$*;*

3. $(1 + \epsilon^{-1}_{jj})D^R_{j\to k} = D_{j\to k}\frac{p_k}{p_j}$ *for* $j \neq k$*, and* $D^R_{j\to j} = D_{j\to j} \equiv -1$*.*

Lemma 1 uses the algebraic definition of diversion ratios and elasticities and is thus independent of the underlying demand model. Lemma 1.1 relates diversion ratios to own-/cross-price elasticities.

[5] First, $\partial R_k/\partial p_j \geq 0$ if j and k are substitutes. Second, $\partial R_j/\partial p_j < 0$ if and only if $\epsilon_{jj} < -1$, which holds in any Bertrand-Nash equilibrium. In sum, $D^R_{j\to k} > 0$. The only exception is the revenue diversion ratio for a self-pair $D^R_{j\to j} \equiv -1 < 0$.

Lemma 1.2 shows how the own-/cross-price elasticities are related. Finally, Lemma 1.3 shows how quantity diversion ratios may be substituted out for revenue diversion ratios.

Lemma 1.3 plays a key role in my identification arguments. The terms $D_{j\to k}\frac{p_k}{p_j}$ enter the firms' optimal pricing equation, gross upward pricing pressure indices, and compensating marginal cost reductions. Conventional approaches assume price and quantity data to calculate them. However, for $j \neq k$,

$$D_{j\to k}\frac{p_k}{p_j} = -\frac{\frac{\partial q_k}{\partial p_j}p_k}{\frac{\partial q_j}{\partial p_j}p_j} = -\frac{\frac{\partial R_k}{\partial p_j}}{\frac{\partial R_j}{\partial p_j} - q_j} = \underbrace{\left(-\frac{\frac{\partial R_k}{\partial p_j}}{\frac{\partial R_j}{\partial p_j}}\right)}_{D^R_{j\to k}}\underbrace{\left(\frac{\frac{\partial R_j}{\partial p_j}}{\frac{\partial R_j}{\partial p_j} - q_j}\right)}_{1+\epsilon_{jj}^{-1}}.$$

That is, the analyst can replace the term $D_{j\to k}\frac{p_k}{p_j}$ with the revenue diversion ratio $D^R_{j\to k}$, multiplied by an "adjustment factor" $(1+\epsilon_{jj}^{-1})$.[6] In the following sections, I show how to estimate revenue diversion ratios and own-price elasticities of demand based on revenue and margin data.

Sum of Diversion Ratios Over Products

It is worth highlighting the summation property of diversion ratios. Whether the diversion ratios sum to one depends on the underlying demand model. In (pure) discrete choice demand models where the total quantity consumed is fixed, quantity diversion ratios from a product to all other alternatives sum to one (Conlon and Mortimer, 2021). However, revenue diversion ratios need not sum to one because the total market revenue may change. Symmetrically, in discrete-continuous choice models where consumers' total budget is fixed, revenue diversion ratios from a product to all other alternatives sum to one. However, quantity diversion ratios need not sum to one because the total units consumed may not be fixed.

3 Identification of Unilateral Effects

In this section, I characterize sufficient conditions for identifying the unilateral effects statistics with the first-order approach developed in Werden (1996), Farrell and Shapiro (2010), and Jaffe and Weyl (2013). I begin with key assumptions on data.

[6] In other words, while the numerator of $D_{j\to k}\frac{p_k}{p_j}$ is $\frac{\partial R_k}{\partial p_j}$, its denominator is $\frac{\partial R_j}{\partial p_j} - q_j$, making it fall short of revenue diversion ratios. The term $(1+\epsilon_{jj}^{-1})$ serves as an adjustment factor that "corrects" the denominator to $\frac{\partial R_j}{\partial p_j}$.

3.1 Assumptions on Data

Assumption 1 (Baseline data). *The analyst observes revenues and relative margins for the merging parties' products.*

Assumption 2 (Revenue diversion ratios data). *The analyst observes the revenue diversion ratios for all pairs of the merging parties' products.*

Assumption 3 (Merger-specific efficiencies). *The analyst knows the percentage decrease in marginal costs for the merging parties' products.*

Assumption 1 requires data on merging firms' product-level revenues and margins but does not require the analyst to observe prices and quantities separately. Assumption 2 departs from the standard approach, which focuses on quantity diversion ratios. I take revenue diversion ratios as given for now, but I show how to estimate them under discrete-continuous demand assumptions in Section 4. Finally, Assumption 3 is standard since estimating merger-specific cost-savings can be quite challenging (Farrell and Shapiro, 2010).

As Farrell and Shapiro (2010) argues, the above assumptions are mild for antitrust authorities who can access merging firms' ordinary course documents and financial data. However, satisfying the above assumptions may not be straightforward in some settings (or necessarily easier than acquiring price/quantity data), especially for researchers without access to confidential data. Nevertheless, the assumptions provide a basis for developing an alternative approach when traditional tools are inapplicable. I provide further discussion on how to overcome measurement issues in Section 3.6.

3.2 Identification of Gross Upward Pricing Pressure Index

Definition

Consider a merger between two firms A and B. Under the unilateral effects theory, the merger shifts the merging firms' pricing incentives upward because they can internalize the opportunity to recapture consumers that would divert to the merger counterparty's products. Farrell and Shapiro (2010) proposes measuring the incentives by *upward pricing pressure (UPP)*, defined as the difference between the pre- and post-merger first-order conditions that are normalized to be quasilinear in marginal cost and evaluated at the pre-merger price (Jaffe and Weyl, 2013).

The primary measure of interest in this paper is the unit-free measure of upward pricing pressure dubbed *gross upward pricing pressure index (GUPPI)*, defined as the upward pricing pressure normalized by the pre-merger price, i.e., $GUPPI_j \equiv UPP_j/p_j$ (Salop and Moresi, 2009; Moresi, 2010). The GUPPI associated with product $j \in \mathcal{J}_A$ can be written as

$$GUPPI_j = \ddot{c}_j(1 - m_j) + \sum_{k \in \mathcal{J}_B} m_k D_{j \to k} \frac{p_k}{p_j}, \tag{4}$$

where $\ddot{c}_j \equiv (c_j^{\text{post}} - c_j^{\text{pre}})/c_j^{\text{pre}} \in (-1, 0)$ represents the percentage decrease in marginal cost from the pre-merger equilibrium due to merger-specific efficiencies.[7] Firm B's GUPPIs are defined symmetrically. When prices are unobserved, economists have often measured GUPPIs by assuming revenues can approximate quantities (e.g., in order to calculate quantity diversion ratios) and that $p_k/p_j \approx 1$ (Ferguson et al., 2023), but such approximations are clearly not innocuous. My identification results show such ad hoc assumptions are unnecessary.

Identification

Using Lemma 1.3, I can rewrite the first-order conditions for an arbitrary firm $F \in \mathcal{F}$ (1) and the gross upward pricing pressure indices for product $j \in \mathcal{J}_A$ (4) as

$$-\epsilon_{jj}^{-1} - m_j + (1 + \epsilon_{jj}^{-1}) \sum_{l \in \mathcal{J}_F \setminus j} m_l D_{j \to l}^R = 0, \tag{5}$$

$$GUPPI_j = \ddot{c}_j(1 - m_j) + (1 + \epsilon_{jj}^{-1}) \sum_{k \in \mathcal{J}_B} m_k D_{j \to k}^R, \tag{6}$$

respectively. If the pre-merger margins and revenue diversion ratios are known, the analyst can use the first-order conditions (5) to identify the own-price elasticities of demand.

Lemma 2 (Identification of own-price elasticities of demand)**.** *Let $F \in \mathcal{F}$ be an arbitrary firm.*

[7] See Online Appendix A for a full derivation. I omit the superscript "pre" when it is clear the object is being evaluated at the pre-merger equilibrium.

For each product $j \in \mathcal{J}_F$,

$$\epsilon_{jj} = -\frac{1 - \sum_{k \in \mathcal{J}_F \setminus j} m_k D^R_{j \to k}}{m_j - \sum_{k \in \mathcal{J}_F \setminus j} m_k D^R_{j \to k}}. \tag{7}$$

Plugging in (7) into (6) to identifies the GUPPIs since they become known functions of merging firms' margins, revenue diversion ratios, and cost savings.[8]

Proposition 1 (Identification of GUPPI)**.** *Under Assumptions 1, 2, and 3, the gross upward pricing pressure indices are identified.*

Unlike the standard approaches, Proposition 1 leverages the firms' profit maximization conditions to identify GUPPI. If the analyst observes prices, costs, and quantity diversion ratios—as assumed in Farrell and Shapiro (2010)—the upward pricing pressures—and thus the GUPPIs—are directly identified, leaving no role for the firms' profit maximization conditions. However, when prices are unobserved, the analyst can leverage the first-order conditions to calculate the merging firms' own-price demand elasticities, which, together with margins and revenue diversion ratios, identify the GUPPIs. My use of firms' profit maximization assumption here reverses the role of margins and price elasticities compared to the more standard approach à la Rosse (1970); rather than using the knowledge of demand price elasticities to recover margins (and thus marginal costs), I use the knowledge of margins to recover the price elasticities.

3.3 Identification of Merger Price Effects

GUPPI statistics have enormous practical virtues because they are simple to calculate, and their signs inform whether prices will rise after a merger (Farrell and Shapiro, 2010). However, translating GUPPIs to merger price effects requires pass-through rates, which quantify how a change in merged firms' opportunity cost impacts their prices at the margin. Based on a first-order approximation argument, Jaffe and Weyl (2013) shows

$$\ddot{p} \approx M \cdot GUPPI, \tag{8}$$

[8] Once the own-price elasticities are identified, the cross-price elasticities can also be recovered using the relationships $D^R_{jk} = -\frac{\epsilon^R_{kj}}{\epsilon^R_{jj}} \frac{R_k}{R_j}$, $\epsilon^R_{jj} = \epsilon_{jj} + 1$, and $\epsilon^R_{kj} = \epsilon_{kj}$.

where $\ddot{p} = (\ddot{p}_j)_{j \in \mathcal{J}_A \cup \mathcal{J}_B}$ is the vector of percentage change in prices due to merger, and M is the *merger pass-through matrix*.[9] Thus, the knowledge of M identifies the merger price effects up to first-order approximation.

Assumption 4 (Pass-through rates)**.** *The analyst observes the merger pass-through rates.*

Proposition 2 (UPP and merger price effects)**.** *Under Assumptions 1, 2, 3, and 4, the merger price effects in percentage terms are identified up to first-order approximation.*

The merger pass-through matrix is calculated from post-merger first-order conditions around the pre-merger equilibrium. However, obtaining reliable estimates of pass-through rates may be non-trivial. For now, I assume the analyst can satisfy Assumption 4 and defer further discussion to Section 3.6.

3.4 Identification of Welfare Effects

I measure a merger's impact on consumer surplus as

$$\Delta CS = \sum_{j \in \mathcal{J}_A \cup \mathcal{J}_B} \Delta CS_j, \tag{9}$$

where each ΔCS_j measures the welfare effect assuming a unilateral price change of p_j holding all else fixed. In other words, measure (9) calculates the total welfare effect as the sum of product-specific welfare effects, each of which assumes a *ceteris paribus* price variation. The additive separability feature ignores externalities or cross-market price effects but allows for a simple calculation.[10] The impact on producer surplus is measured analogously.

Figure 1 visualizes a scenario where the price of a product increases from p_j^0 to p_j^1 so the quantity demanded decreases from q_j^0 to q_j^1. The product also enjoys a marginal cost reduction from c_j^0 to c_j^1. The figure shows that first-order approximations of the associated impacts on consumer surplus

[9]Note that my merger pass-through rates differ slightly from Jaffe and Weyl (2013)'s since I use GUPPIs instead of UPPs. Let Λ be a diagonal matrix of $1/p_j$'s. Jaffe and Weyl (2013) shows $\Delta p \approx M^* \cdot UPP$. Since $GUPPI = \Lambda UPP$ and $\ddot{p} = \Lambda \Delta p$, we have $\ddot{p} = \Lambda \Delta p = \Lambda M^* UPP = \Lambda M^* \Lambda^{-1} \Lambda UPP = \Lambda M^* \Lambda^{-1} GUPPI$, so $M = \Lambda M^* \Lambda^{-1}$.

[10]Computing consumer surplus variation following multiple price changes can be difficult if the integral is not path-independent (Chipman and Moore, 1980). In practice, it is common to approximate the total welfare effects as the sum of product-specific welfare effects, each of which assumes a *ceteris paribus* price change (Araar and Verme, 2019).

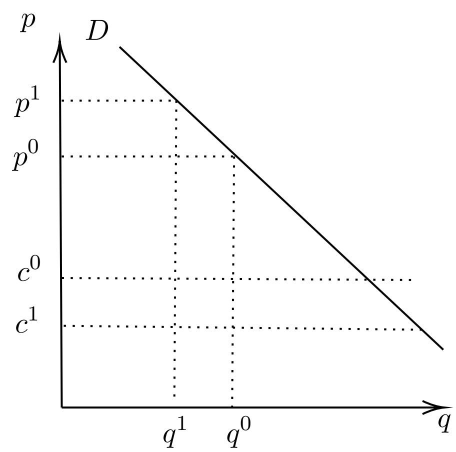


Figure 1: Calculating welfare effects from a merger

and producer surplus give

$$\Delta CS_j \approx -(\Delta p_j \times q_j^0 + \frac{1}{2} \times \Delta p_j \times \Delta q_j),$$

$$\Delta PS_j \approx \Delta p_j \times q_j^1 + \Delta q_j \times (p_j^0 - c_j^0) - \Delta c_j \times q_j^1.$$

The impact on consumer surplus is approximated using the trapezoid defined by p_j^0 and p_j^1 and the residual demand function.[11] The impact on producer surplus is the sum of gained revenue ($\Delta p_j \times q_j^1$), lost revenue ($\Delta q_j \times (p_j^0 - c_j^0)$), and cost savings ($-\Delta c_j \times q_j^1$). The impact on social surplus is their sum. The following lemma shows that the analyst can calculate ΔCS_j and ΔPS_j using revenue, margin, own-price elasticity, the percentage increase in price, and the cost-efficiency savings, all of which are identified or known under the previous assumptions.

Lemma 3 (Approximation of welfare effects)**.** *The changes in consumer surplus and producer surplus associated with a price increase of product j are approximately*

$$\Delta CS_j \approx -\ddot{p}_j R_j (1 + \frac{1}{2}\epsilon_{jj}\ddot{p}_j),$$

$$\Delta PS_j \approx \ddot{p}_j R_j (1 + \epsilon_{jj}\ddot{p}_j) + \epsilon_{jj} R_j \ddot{p}_j m_j - \ddot{c}_j (1 - m_j) R_j (1 + \epsilon_{jj}\ddot{p}_j).$$

Proposition 3 (Identification of welfare effects)**.** *Under Assumptions 1, 2, 3, and 4, the welfare*

[11] Compensating variations and equivalent variations are equal to consumer surplus variation up to first-order approximation with price change of a single product (Willig, 1976). The three measures are identical when there is no income effect.

effects of a merger—measured in consumer surplus and producer surplus—are identified up to first-order approximation.

Remark 1 (Identifying bounds on consumer welfare effects)**.** Laspeyres variation and Paasche variation are other measures of consumer welfare that are useful and easy to interpret. Laspeyres variation, defined as $LV_j \equiv -\Delta p_j \times q_j^0$, represents the change in income necessary to purchase the same bundle of goods purchased before the price variation. Paasche variation, defined as $PV_j \equiv -\Delta p_j \times q_j^1$, represents the change in income required to purchase the final bundle of goods at initial prices. They are useful because they provide bounds on consumer welfare variation: If good j is normal, under standard regularity conditions, $LV_j < CV_j < \Delta CS_j < EV_j < PV_j < 0$ following an increase in p_j. It is straightforward to verify that $LV_j = -\ddot{p}_j R_j$ and $PV_j \approx -(1 + \epsilon_{jj}\ddot{p}_j)\ddot{p}_j R_j$. Thus, Laspeyres and Paasche variations are also identified under the same conditions as Proposition 3. In Online Appendix B, I show that compensating variation can be calculated exactly (i.e., without approximation) under CES utility.

3.5 Identification of Compensating Marginal Cost Reductions

Compensating marginal cost reductions (CMCR) are defined as the percentage decrease in the merging parties' costs that would leave the pre-merger prices unchanged after the merger. In contrast to upward pricing pressures that assume other products' marginal costs are fixed, compensating marginal cost reductions capture the effects of simultaneous changes in marginal costs.[12] Werden (1996) shows that compensating marginal cost reductions are identified when prices, margins, and quantity diversion ratios are observed. I show that compensating marginal cost reductions are identified under a weaker data requirement. The following assumption defines compensating marginal cost reductions.

Assumption 5 (CMCR)**.** *A merger reduces the marginal costs of the merging parties' products up to the level that leaves the prices unchanged.*

Let $\ddot{c}_j = (c_j^1 - c_j^0)/c_j^0$ denote the percentage reduction in marginal costs, where c_j^1's are the post-merger marginal costs determined by Assumption 5.

[12] Upward pricing pressure can be more conservative because the value of sales diverted to the merging counterparty is larger if profit margins increase due to a reduction in marginal costs. Farrell and Shapiro (2010) endorses upward pricing pressure on the grounds of simplicity and transparency while acknowledging that compensating marginal cost reductions can be more accurate.

Lemma 4 (CMCR)**.** *Suppose the post-merger marginal costs satisfy Assumption 5. The corresponding compensating marginal cost reductions for each product j is equal to*

$$\ddot{c}_j = \frac{m_j^0 - m_j^1}{1 - m_j^0}, \tag{10}$$

where the post-merger margins m_j^1's are implicitly defined by the post-merger first-order conditions

$$-\epsilon_{jj}^{-1} - m_j^1 + (1 + \epsilon_{jj}^{-1}) \sum_{l \in \mathcal{J}_A \backslash j} m_l^1 D_{j \to l}^R + (1 + \epsilon_{jj}^{-1}) \sum_{k \in \mathcal{J}_B} m_k^1 D_{j \to k}^R = 0. \tag{11}$$

The analyst can calculate the compensating marginal cost reductions in two steps. First, identify the post-merger margins implied by Assumption 5 using the post-merger first-order conditions (11), which constitutes a system of linear equations. Note that the own-price demand elasticities and revenue diversion ratios in (11) correspond to the pre-merger levels as prices are assumed to remain fixed after the merger. Second, given the pre- and post-merger margins, compute the percentage change in marginal costs from the pre-merger equilibrium via (10).

Proposition 4 (Identification of CMCR)**.** *Under Assumptions 1, 2, and 5, the compensating marginal cost reductions are identified.*

The data requirement of Proposition 4 is weaker than that of Werden (1996); Werden (1996)'s assumption that the analyst observes price, quantity, margin, and quantity diversion ratios is sufficient to identify revenue, margin, and revenue diversion ratios, satisfying the conditions of Proposition 4.

3.6 Measurement Issues

My approach bypasses the standard price/quantity data requirement but assumes access to merging firms' revenues, margins, and revenue diversion ratios. Furthermore, translating upward pricing pressure statistics to predictions on merger price effects requires knowledge of pass-through rates. While satisfying these assumptions may be non-trivial (or even more difficult than acquiring price/quantity data), there are alternative ways to meet the data assumptions, as I discuss below. I note that while measurement problems are prevalent in antitrust cases, my framework facilitates rigorous sensitivity analysis that allows analysts to determine whether the conclusion is robust in a reasonable range of measurement errors.

Measuring Margins

One way to measure firms' margins is to use their financial data. During merger investigations, antitrust authorities and courts can access firms' data for margin calculation. It is well known that using accounting data to infer margins can be tricky (see, e.g., Fisher and McGowan (1983) and Sacher and Simpson (2020)). However, as Farrell and Shapiro (2010) also argues, antitrust agencies can often measure margins reasonably well. Researchers may also use information from public filings or previous research; see, e.g., Smith (2004) and Ellickson et al. (2020) for supermarket industry examples.

An alternative approach is to take the production function approach to markup estimation. The literature has shown that markups may be estimated without output price data.[13] The downside of this approach is twofold. First, it may be difficult to access specific firms' production data. Second, when firms are producers of multiple products, estimating product-level margins may be more difficult relative to estimating firm-level margin (Orr et al., 2024). Yet, when product-level margins are unavailable, firm-level margins can also be informative. In particular, when consumers have CES preference, firms charge uniform relative margins over their products, so firm-level margins should be close to product-level margins (Nocke and Schutz, 2018).

Measuring Revenue Diversion Ratios

There are several ways to measure revenue diversion ratios. First, estimates of revenue diversion ratios may be obtained from merging firms' internal documents as firms often conduct revenue leakage or consumer share of wallet analysis.[14] Second, revenue diversion ratios may be estimated from consumer surveys that ask how consumers would re-allocate their spending following hypothetical price or product availability scenarios (Conlon and Mortimer, 2021). Third, the analyst may conduct a reduced-form or structural econometric analysis based on the availability of exogenous

[13] See, e.g., Klette and Griliches (1996), De Loecker (2011), De Loecker and Warzynski (2012), Grieco et al. (2016), Gandhi et al. (2020), Kasahara and Sugita (2020), De Ridder et al. (2022), Raval (2023), and Kirov et al. (2023).

[14] If firms more commonly record performance in revenues than in quantities sold, then revenue diversion ratios may be easier to estimate than quantity diversion ratios from company documents.

identifying events (e.g., entries and exits of products). Finally, the analyst may estimate revenue diversion ratios from cross-sectional consumer expenditure data based on discrete-continuous demand assumption, which is what I show in the next section.

Measuring Pass-through Rates

One of the main advantages of the first-order approach derives from the ability to express merger price and welfare effects as functions of sufficient statistics that may be estimated from various methods. However, merger pass-through rates can be challenging to estimate because they depend on demand curvature.[15] In particular, estimating a merger pass-through matrix via a reduced-form regression of price on cost may not work well in practice (MacKay et al., 2014; Miller et al., 2016, 2017a; Miller and Sheu, 2021). Moreover, the dimensionality of the pass-through matrix increases with the number of merging firms' products.

Fortunately, Miller et al. (2017b) and Dutra and Sabarwal (2020) present simulation evidence that upward pricing themselves are often good proxies for the true merger price effects. In other words, the analyst can simply assume $M \approx I$ and approximate the percentage change in price as

$$\ddot{p}_j \approx GUPPI_j \tag{12}$$

The assumption substantially relaxes the methodological and data requirement for approximating the true merger price effects. An analyst who does not have information on demand curvature—regardless of whether price data are accessible—would follow Miller et al. (2017b)'s recommendation.[16] The analyst can refine the estimates whenever further information on the pass-through rates is available. In Online Appendix C, I provide simulation evidence that upward pricing pressure is a conservative predictor of the true merger price effect when the demand is CES, which complements Miller et al. (2017a)'s findings.

[15] An alternative to using the merger pass-through matrix is to use the (pre-merger) cost pass-through matrix based on pre-merger first-order conditions around the pre-merger equilibrium (Farrell and Shapiro, 2010); Jaffe and Weyl (2013) notes that the choice between merger and cost pass-through rates may not be empirically significant. Miller et al. (2016) describes conditions for identifying merger pass-through rates based on pre-merger pass-through rates.

[16] Miller et al. (2017b) also documents simulation evidence that shows prediction error from using identity matrix in place of true merger pass-through matrix does not systematically exceed misspecification error. In other words, using pass-through rates estimated from a misspecified model may perform worse than simply treating upward pricing pressures as predictors of merger price effects.

Finally, imposing functional form assumptions on demand can facilitate the identification of merger pass-through matrices. In Online Appendix D, I derive a merger pass-through matrix assuming a single representative consumer with CES preference. I show that the analyst can calculate the merger pass-through matrix using only the merging firms' revenues and margins. I apply this approach in my empirical application in Section 6.

4 Consumer Demand and Revenue Diversion Ratios

In this section, I show that a discrete-continuous demand assumption facilitates the identification and estimation of revenue diversion ratios.

4.1 Setup

I specify consumer-side model primitives as a tuple $\langle \mathcal{I}, (\mathcal{C}_i, U_i, B_i)_{i\in\mathcal{I}}\rangle$, where $\mathcal{I}$ is the set of consumers, $\mathcal{C}_i$ is consumer i's consideration set, $U_i : \mathbb{R}^{\dim(\mathcal{C}_i)} \to \mathbb{R}$ is consumer i's utility function, and $B_i \in \mathbb{R}_+$ is i's budget.[17] I assume consideration sets always include an outside option $j = 0$. To simplify the exposition, I assume that the analyst knows every consumer's consideration set $\mathcal{C}_i$ and budget B_i, although the analyst may need to estimate them in practice.

4.2 Consumer's Problem

Each consumer $i \in \mathcal{I}$ maximizes utility U_i with respect to consumption vector $q_i \equiv (q_{ij})_{j\in\mathcal{C}_i}$ subject to a budget constraint $\sum_{j\in\mathcal{C}_i} p_j q_{ij} \leq B_i$. The Marshallian demand function induces consumer i's expenditure on each product j as

$$e_{ij} = \alpha_{ij} B_i, \tag{13}$$

where $\alpha_{ij} \in [0, 1]$ represents the expenditure share of budget allocated to product j. Product j's revenue is the sum of consumers' expenditure on the product,

$$R_j = \int_{i\in\mathcal{S}_j} e_{ij} d\mu, \tag{14}$$

[17] Consumers' consideration sets can be dropped from the model primitives without loss of generality. However, it is conceptually and computationally helpful to make them explicit. In my second empirical application, consumers' consideration sets are determined by the distance from their residences to grocery stores.

where $i \in \mathcal{S}_j$ (i is a shopper of j) if and only if $j \in \mathcal{C}_i$ (i considers j), and μ is a measure on the set of consumers.

For instance, Ellickson et al. (2020) estimates a spatial demand model to analyze grocery store competition. Each $i \in \mathcal{I}$ represents a census tract, and each $j \in \mathcal{J}$ represents a grocery store. The authors assume that consumers consider all grocery stores within 15 miles of their location, which pins down $\mathcal{C}_i$. The grocery budget B_i is a constant fraction of the tract's aggregate income, which the authors estimate to be around 13%. Measure μ is a counting measure, so (14) reduces to $R_j = \sum_{i \in \mathcal{S}_j} e_{ij}$, i.e., the total revenue of each store is the sum of tract-level expenditures on that store.

4.3 Identification of Revenue Diversion Ratios

Plugging in (14) to the definition of revenue diversion ratio (3) allows me to express the diversion ratio as a weighted sum of consumer-level diversion ratios.[18]

Lemma 5 (Revenue diversion ratio as a weighted sum)**.** *Assuming that Leibniz's rule applies,*

$$D^R_{j \to k} = \int_{i \in \mathcal{S}_k} w_{ij} D^R_{i,j \to k} d\mu, \tag{15}$$

where

$$\begin{aligned} w_{ij} &= \frac{\partial e_{ij}/\partial p_j}{\int_{\tilde{i} \in \mathcal{S}_j} \partial e_{\tilde{i}j}/\partial p_j d\mu}, \\ D^R_{i,j \to k} &= -\frac{\partial e_{ik}/\partial p_j}{\partial e_{ij}/\partial p_j}. \end{aligned}$$

Consumers whose expenditures are more sensitive to price receive higher weights. At first glance, (15) requires prices to be observed. However, I can relax the data requirement by imposing the following additive random utility model assumption motivated by the discrete-continuous demand literature.

Assumption 6 (Latent utility)**.** *Let $u^*_{ij} = u_{ij} + \varepsilon_{ij}$ be the latent utility of consumer i from consuming product j for each unit of expenditure, where u_{ij} is the deterministic component and ε_{ij}*

[18] A quantity diversion ratio analog appears in Hosken and Tenn (2016).

the random component of latent utility. Assume that the latent utility from the outside option is normalized to $u_{i0} = 0$.

1. *For each consumer* i *and product* j, $\alpha_{ij} = \mathbb{E}\left[\mathbb{I}\{j = \arg\max_{l \in \mathcal{C}_i} u^*_{il}\}\right]$.

2. *For each consumer* i *and product* j, $u_{ij} = \delta_{ij} + (1 - \eta) \log p_j$, *where* δ_{ij} *is the latent utility from non-price characteristics of product* j, *and* $\eta > 1$ *is the price responsiveness parameter.*

Assumption 6.1 allows the analyst to treat consumers as if they were making a discrete choice based on an additive random utility model for every unit of their budgets. For example, as I show below in Section 4.4, a constant elasticity of substitution utility function assumption renders $\alpha_{ij} = u_{ij}/(\sum_{l \in \mathcal{C}_i} u_{il})$, i.e., consumer i's expenditure share on product j takes the familiar softmax form. Assumption 6.2 has three roles. First, it imposes an exclusion restriction that p_j only affects consumer i's expenditure through u_{ij}. Second, it assumes homogeneous price responsiveness to simplify the identification problem.[19] Third, it assumes log-linearity in price, which can facilitate merger simulation.[20] In sum, Assumption 6 admits a large class of discrete-continuous choice additive random utility models (e.g., nested CES demand) but rules out models in which consumers have heterogeneous price sensitivity (e.g., random coefficient models).

Dubé et al. (2022) provides a microeconomic foundation for Assumption 6. Suppose consumer i has perfect substitute preference whose (constant) marginal utility over each product depends on the random utility shock ε_{ij}. The perfect substitute preference induces the individual to make a discrete choice over products as consuming at most one good is optimal. Taking expectation over the discrete-continuous demand over ε_i leads to a demand described by Assumption 6. Note that the mean utility function u_{ij}'s are different from the primitive utility function U_i, which does not depend on ε_i—the demand system derived from U_i corresponds to the *expected* demand system derived from the discrete choice additive random utility assumption.

[19] Assuming consumers have homogeneous price sensitivity is restrictive, but it is possible to relax this assumption. For example, suppose $u_{ij} = \delta_{ij} + \kappa_i \log p_j$, and $\kappa_i = \kappa_0 + \kappa_1 x_i$, where x_i is some observable scalar covariate that affects consumers' sensitivity to price. Then, the analyst may estimate κ_0 and κ_1 by finding values that best match the own-price elasticities implied by observed margins. However, estimation with standard random coefficient assumptions appears challenging without further data/modeling assumptions and is beyond the scope of this paper.

[20] Log-linearity is not strictly necessary for establishing the identification arguments for revenue diversion ratios. However, such specification is standard. Furthermore, Dubé et al. (2022) shows that log-linearity is crucial for establishing the Hurwicz-Uzawa integrability of the demand system.

Lemma 6 (Revenue diversion ratios with latent utility)**.** *Assumption 6 implies*

$$w_{ij} = \frac{(\partial \alpha_{ij}/\partial u_{ij})B_i}{\int_{\tilde{i}\in\mathcal{S}_j}(\partial \alpha_{\tilde{i}j}/\partial u_{\tilde{i}j})B_{\tilde{i}}d\mu},$$
$$D^R_{i,j\to k} = -\frac{\partial \alpha_{ik}/\partial u_{ij}}{\partial \alpha_{ij}/\partial u_{ij}}.$$

Lemma 6 shows that computing $D^R_{j\to k}$ only requires finding the sensitivity of consumers' expenditure shares with respect to their mean utility for product j at the margin. I provide sufficient conditions to identify revenue diversion ratios as follows.

Assumption 7 (Hotz-Miller inversion)**.** *Let H_i be the distribution of preference shock $\varepsilon_i = (\varepsilon_{ij})_{j\in\mathcal{C}_i}$.*

1. *Distribution H_i is absolutely continuous with full support on $\mathbb{R}^{\dim(\mathcal{C}_i)}$.*

2. *The analyst observes $\alpha_i = (\alpha_{ij})_{j\in\mathcal{C}_i}$ for all i.*

3. *The analyst knows the distribution H_i.*

Assumption 7 lays out the classical conditions for applying Hotz and Miller (1993)'s inversion theorem, which says that the mapping from u_i to α_i is invertible.[21] Since the analyst knows the inverse mapping from α_i to u_i as well as the pre-merger values of α_i, the analyst can evaluate the partial derivatives that appear in Lemma 6.

Proposition 5 (Identification of revenue diversion ratios with latent utility)**.** *Under Assumptions 6 and 7, the revenue diversion ratios across all pairs of merging firms' products are identified.*

While the above proposition assumes knowledge of budgets, expenditure shares, and the distribution of preference shocks, it is also common to estimate them using parametric assumptions, i.e., let $B_i = B_i^\theta$, $\alpha_i = \alpha_i^\theta$, and $H_i = H_i^\theta$, and estimate θ using available moment conditions. The feasibility of this strategy relies on the availability of a rich set of covariates that can control for the effects of unobserved prices. For instance, in Ellickson et al. (2020), the authors assume that (i) B_i^θ is a constant fraction of total income, (ii) u_{ij}^θ is a linear function of distance and a rich set of tract and store characteristics and their interactions, and (iii) α_{ij}^θ follows a nested logit model,

[21] I consider Hotz and Miller (1993)'s classical assumptions since they hold in most applications. However, recent developments allow for weaker assumptions; see, e.g., Chiong et al. (2016), Galichon (2018), and Sørensen and Fosgerau (2022).

where the nests are defined by grocery store formats. They then estimate the parameters using store-level revenue data and the moment condition (14). The authors control for unobserved prices using chain indicators, citing recent empirical findings that prices tend to be uniform across stores within a chain (DellaVigna and Gentzkow, 2019; Hitsch et al., 2021).

4.4 CES Utility Function

Assuming that consumers have a constant elasticity of substitution utility function (Spence, 1976; Dixit and Stiglitz, 1977) further simplifies the econometric problem.

Assumption 8 (CES utility)**.** *Consumers' preferences can be described by a constant elasticity of substitution (CES) function*

$$U_i(q_i) = A\left(\sum_{j\in\mathcal{C}_i} \beta_{ij}^{\frac{1}{\eta}} q_{ij}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \tag{16}$$

with parameters $A, \eta, \beta_{ij} > 0$.[22]

Lemma 7 (Revenue diversion ratios under CES utility)**.** *Under Assumption 8, the fraction of consumer i's budget spent on each product* $j \in \mathcal{C}_i$ *is*

$$\alpha_{ij} = \frac{\exp(u_{ij})}{\sum_{k\in\mathcal{C}_i}\exp(u_{ik})}, \tag{17}$$

where $u_{ij} \equiv \log\beta_{ij} + (1-\eta)\log p_j$*. Furthermore, the weights and consumer-level diversion ratios are*

$$w_{ij} = \frac{\alpha_{ij}(1-\alpha_{ij})B_i}{\int_{\tilde{i}\in\mathcal{S}_j}\alpha_{\tilde{i}j}(1-\alpha_{\tilde{i}j})B_{\tilde{i}}d\mu}, \tag{18}$$

$$D^R_{i,j\to k} = \frac{\alpha_{ik}}{1-\alpha_{ij}}, \tag{19}$$

respectively.

Lemma 7 shows consumers with CES preferences determine their expenditure shares as if they were making discrete choices with preference shocks that independently follow the Type-1 Extreme

[22] Parameter A is an arbitrary scaling factor (which I do not use), η is the elasticity of substitution between products, and β_{ij} is the product j's quality perceived by consumer i. Note that (i) if $\eta\to\infty$, $U_i(q_i)\to A\sum_{j\in\mathcal{C}_i} q_{ij}$; (ii) if $\eta\to 1$, $U_i(q_i)\to (A/\prod_{j\in\mathcal{C}_i}\beta_{ij}^{\beta_{ij}})\prod_{j\in\mathcal{C}_i} q_{ij}^{\beta_{ij}}$; and (iii) if $\eta\to 0$, $U_i(q_i)\to A\min_{j\in\mathcal{C}_i}\{q_{ij}/\beta_{ij}\}$.

Value distribution. Furthermore, it shows that the analyst needs consumers' expenditure shares *only* for the merging firms' products to calculate the diversion ratios, allowing Assumption 7 (which requires the analyst to observe the expenditure shares on *all* products) to be relaxed. If there is a single representative consumer in the market, then the diversion ratio formula collapses to $D^R_{j \to k} = \frac{\alpha_k}{1-\alpha_j}$, so the merging firms' product-specific market shares are sufficient to identify their revenue diversion ratios.

Assumption 9 (Consumer expenditure share data)**.** *The analyst observes consumers' expenditure shares on the merging firms' products.*

Proposition 6 (Identification of revenue diversion ratios under CES preference)**.** *Under Assumptions 6, 8, and 9, the revenue diversion ratios across all pair of products of the merging firms are identified.*

Proposition 6 is powerful since it only requires data from the merging parties. For example, the researcher may use merging firms' transaction data (e.g., credit card or loyalty program data) to estimate consumers' expenditure shares on the merging parties' products (Hosken and Tenn, 2016). Alternatively, one may use simple consumer surveys to solicit information on the total budget consumers allocate to the relevant products and their expenditures on the merging firms' products (Reynolds and Walters, 2008). If there is a single representative agent, then Proposition 6 only requires the total market size and the merging firms' revenues for each of their products; my first empirical application in Section 6 exploits this result.

Another advantage of assuming CES preference is that it allows the analyst to measure revenue diversion ratios using second-choice data.[23] Proposition 7 justifies studying intertemporal variations in revenues following a removal of a product when the analyst is unable to observe competitors' revenues, which are inputs to revenue shares calculation.

Proposition 7 (Identification from second choice data)**.** *Under Assumption 8, the revenue diversion ratio from product j to product k following a removal of product j is equal to the revenue diversion ratio derived from a marginal increase in product j's price.*

Assuming CES preference permits analytic tractability and has low data requirement. However, as with logit demand, CES demand imposes a restrictive substitution pattern as the cross-price

[23] A quantity diversion ratio analog appears in Conlon and Mortimer (2021).

elasticity ϵ_{kj} only depends on index j but not k (Nevo, 2011; Berry and Haile, 2021; Conlon and Mortimer, 2021). Thus, choosing CES demand function may be inappropriate if the analyst's objective is to explain micro data and discover rich substitution patterns.[24] A mixed CES specification has more plausible range of elasticities and thus can yield more realistic average predicted price effects (Björnerstedt and Verboven, 2016) but is more demanding on data. In my empirical application that studies the merger between Albertson's and Safeway, I use a nested CES preference assumption to balance tractability and realism in consumer substitution patterns.

5 Merger Simulation

I show that merger simulation is feasible without price data when consumers have discrete-continuous preferences with homogeneous price responsiveness as described in Assumption 6. Specifically, although it is impossible to predict the post-merger prices p_j^{post} or the difference in pre- and post-merger prices $\Delta p_j = p_j^{\text{post}} - p_j^{\text{pre}}$, it is possible to predict the percentage change in price relative to the pre-merger equilibrium $\ddot{p}_j = (p_j^{\text{post}} - p_j^{\text{pre}})/p_j^{\text{pre}}$. Thus, the proposed methodology allows the analyst to make statements such as "In the post-merger equilibrium, the prices of products 1, 2, and 3 will be 3%, 6%, and 10% higher, respectively, relative to the pre-merger equilibrium."

Since a merger changes the ownership structure and possibly the marginal costs of the merging firms, it shifts the merging firms' first-order condition to

$$-\epsilon_{jj}^{-1} - m_j + (1 + \epsilon_{jj}^{-1}) \sum_{l \in \mathcal{J}_A \setminus j} m_l D_{j \to l}^R + (1 + \epsilon_{jj}^{-1}) \sum_{k \in \mathcal{J}_B} m_k D_{j \to k}^R = 0,$$

while the first-order conditions of the non-merging firms remain as

$$-\epsilon_{jj}^{-1} - m_j + (1 + \epsilon_{jj}^{-1}) \sum_{l \in \mathcal{J}_F \setminus j} m_l D_{j \to l}^R = 0.$$

The key idea is to solve the post-merger first-order conditions after re-expressing them as a known function of percentage deviation in prices $(\ddot{p}_j)_{j \in \mathcal{J}}$. The following lemma shows how the post-merger objects are related to an arbitrary vector of $(\ddot{p}_j)_{j \in \mathcal{J}}$.

[24] Yet, Head and Mayer (2023) provides simulation evidence that CES demand specification can provide close approximations to the predictions of more complex BLP-type demand models for measuring aggregate outcomes.

Lemma 8 (Merger simulation)**.** *Suppose Assumption 6 holds. Let $\ddot{p}$ be an arbitrary vector of $\ddot{p}_j = (p_j^{post} - p_j^{pre})/p_j^{pre}$. For all consumer i and product j,*

$$
\begin{aligned}
u_{ij}^{post} &= u_{ij}^{pre} + (1-\eta)\log(1+\ddot{p}_j), \\
\alpha_{ij}^{post} &= \mathbb{E}[\mathbb{I}\{j = \arg\max_{l\in\mathcal{C}_i}(u_{il}^{post} + \varepsilon_{il})\}], \\
R_j^{post} &= \int_{i\in\mathcal{S}_j} \alpha_{ij}^{post} B_i d\mu, \\
\epsilon_{jj}^{R,post} &= \frac{1-\eta}{R_j^{post}} \int_{i\in\mathcal{S}_j} (\partial\alpha_{ij}^{post}/\partial u_{ij}^{post}) B_i d\mu, \\
\epsilon_{jj}^{post} &= \epsilon_{jj}^{R,post} - 1, \\
D_{j\to l}^{R,post} &= -\frac{\int_{i\in\mathcal{S}_l}(\partial\alpha_{il}^{post}/\partial u_{ij}^{post})B_i d\mu}{\int_{i\in\mathcal{S}_j}(\partial\alpha_{ij}^{post}/\partial u_{ij}^{post})B_i d\mu}, \\
m_j^{post} &= 1 - (1-m_j^{pre})\left(\frac{1+\ddot{c}_j}{1+\ddot{p}_j}\right).
\end{aligned}
$$

Lemma 8 clarifies how the percentage deviation in price $(\ddot{p}_j)_{j\in\mathcal{J}}$ cascades to the objects that appears in the post-merger first-order conditions. The following assumptions are sufficient to calculate the vector $(\ddot{p}_j)_{j\in\mathcal{J}}$ that solves the post-merger first-order conditions.

Assumption 10 (Merger simulation)**.** *The analyst knows pre-merger mean utilities $(u_{ij}^{pre})_{i\in\mathcal{I},j\in\mathcal{J}}$, margins $(m_j^{pre})_{j\in\mathcal{J}}$, percentage reductions in marginal costs $(\ddot{c}_j)_{j\in\mathcal{J}}$, the price elasticity parameter η, and the distribution of $\varepsilon_i = (\varepsilon_{ij})_{j\in\mathcal{J}}$.*

It is straightforward to verify that the post-merger first-order conditions are known functions of $\ddot{p}$ under Assumptions 6 and 10. Thus, it is possible to solve for $\ddot{p}$ that satisfies the post-merger first-order conditions.

Proposition 8 (Merger simulation)**.** *Under Assumptions 6 and 10, the percentage price changes from the pre-merger equilibrium to the post-merger equilibrium for all products in the market are identified.*

I close the section by discussing how to meet Assumption 10 with data on consumer expenditure and merging firms' margins.

Corollary 1 (Merger simulation)**.** *Suppose Assumptions 3, 6, and 7 hold, and the analyst observes merging firms' margins. The percentage price changes from the pre-merger equilibrium to the post-merger equilibrium for all products in the market are identified.*

Recall that Assumption 3 says post-merger efficiency credits $\ddot{c}_j$'s are known; Assumption 6 describes expenditure shares as functions of mean utilities; Assumption 7 describes conditions for inverting the mapping from mean utilities to consumer expenditure shares. If the analyst knows the distribution of ε_i and the pre-merger expenditure shares $(\alpha_{ij})_{i\in\mathcal{I},j\in\mathcal{J}}$, then Hotz-Miller inversion identifies the pre-merger mean utility $(u_{ij}^{\text{pre}})_{i\in\mathcal{I},j\in\mathcal{J}}$. Next, given that $\epsilon_{jj}^R = \frac{1-\eta}{R_j}\int_{i\in\mathcal{S}_j}(\partial\alpha_{ij}/\partial u_{ij})B_i d\mu$ and $\epsilon_{jj} = \epsilon_{jj}^R - 1$, parameter η can be (over-)identified from data on merging firms' margins. Once η is identified, then data on pre-merger expenditure shares and consumer budget identifies the own-price elasticity $(\epsilon_{jj}^{\text{pre}})_{j\in\mathcal{J}}$ of all products in the market. The pre-merger own-price elasticities, pre-merger margins, and pre-merger revenue diversion ratios identify $(m_j^{\text{pre}})_{j\in\mathcal{J}}$ from the pre-merger first-order conditions (5). In sum, to run merger simulation, it is sufficient that the analyst (i) knows (or estimates) the distribution of random utility shocks, (ii) observes (or estimates) consumer expenditures on all products, and (iii) observes merging firms' margins.

6 Empirical Application I: Staples/Office Depot (2016)

I apply my framework to evaluate the proposed merger of Staples and Office Depot, which was eventually blocked in 2016. I use this empirical example to illustrate the simplicity of my approach.

6.1 Background

Staples and Office Depot are the two largest suppliers of office supplies and services in the United States. On February 4, 2015, Staples entered into a \$6.3 billion merger agreement with Office Depot. On December 9, 2015, the Federal Trade Commission (FTC) sued to block the merger over concerns that it would significantly reduce competition nationwide in the market for consumable office supplies sold to large business customers.[25] On May 10, 2016, a federal judge granted the FTC's preliminary injunction to block the merger. The parties subsequently announced that they

[25] The FTC defined large business customers as those that purchased at least \$500,000 worth of consumable office supplies during 2014 (United States District Court for the District of Columbia, 2016).

would abandon the deal.

The claimed market constitutes a cluster market for consumable office supplies that are not substitutes for each other (e.g., pens, file folders, Post-it notes, binder clips, papers, etc.). Defining the market as a cluster market was justified as "market shares and competitive conditions are likely to be similar for the distribution of pens to large customers and the distribution of binder clips to large customers" (Shapiro, 2016), to which the court agreed. Obtaining reliable price data for many items that enter the cluster market or computing the relevant price indices is challenging. I show that with the proposed methodology, I can easily calculate the merger price effects, welfare effects, and compensating marginal cost reductions.

6.2 Model

I assume that the competition in the consumable office supplies markets for large B-to-B customers can be approximated by the standard Bertrand Nash framework. However, the market has features of multiple models, especially bargaining and auction models, since competition for large B-to-B customers often occurs through formal or informal bidding processes for core products.[26] To focus on demonstrating how the proposed methodology can be applied in a real-world example, I abstract away from detailed institutional features and assume that competition in posted price is a reasonable approximation. I assume a single representative consumer with CES preference.

6.3 Data

Since I cannot access the confidential data used in the investigation, I use publicly available documents to infer the parties' revenues, margins, and shares. Based on the available case materials, I infer the total market size in 2014 to be $B = \$2.05$ billion and the revenue shares of Staples and Office Depot to be $\alpha_{\mathrm{SP}} = 47.3\%$ and $\alpha_{\mathrm{OD}} = 31.6\%$, respectively.[27] Based on the companies' 10-K

[26] See the FTC's proposed findings of fact submitted to the court, available at https://www.ftc.gov/system/files/documents/cases/160420staplesfindings.pdf. Specifically, for core products, large B-to-B customers solicit pricing and service information from prospective office supplies vendors through requests for proposals or similar processes. For non-core products, large B-to-B customers generally pay a flat percentage discount off published prices. Large B-to-B customers' contracts for consumable office supplies contain additional financial incentives beyond low pricing, such as volume-based rebates or signing bonuses.

[27] I use the presentation material prepared by the FTC's economic expert Carl Shapiro (see pp.23-27 of Shapiro (2016)). The information is also available from the Memorandum Opinion on FTC v. Staples, Inc. and Office Depot, Inc. (United States District Court for the District of Columbia, 2016).

documents for fiscal year 2014, I infer Staples' and Office Depot's margins to be $m_{\text{SP}} = 25.8\%$ and $m_{\text{OD}} = 23.4\%$, respectively.[28]

6.4 Results

Gross Upward Pricing Pressure Indices and Consumer Harm

I estimate the annual consumer harm as follows. The margins imply own-price elasticities of $\epsilon_{\text{SP,SP}} = -3.875$ and $\epsilon_{\text{OD,OD}} = -4.273$. The market shares imply revenue diversion ratios of $D^R_{\text{SP}\to\text{OD}} = 59.9\%$ and $D^R_{\text{OD}\to\text{SP}} = 69.1\%$. The gross upward pricing pressure indices, in the absence of cost efficiency credits, are $GUPPI_{\text{SP}} = 10.4\%$ and $GUPPI_{\text{OD}} = 13.7\%$. In Online Appendix D, I show that I can calculate the merger pass-through matrix using data on merging firms' revenues and margins under the CES preference assumption. The estimated merger pass-through matrix is

$$M = \begin{bmatrix} 1.005 & 0.345 \\ 0.347 & 1.098 \end{bmatrix}.$$

The first-order merger price effects, calculated as $\ddot{p} \approx M \cdot GUPPI$, are $\ddot{p}_{SP} = 15.2\%$ and $\ddot{p}_{OD} = 18.7\%$.[29] The estimated annual consumer harm, calculated using the expression in Lemma 3, is \$177 million.

Compensating Marginal Cost Reductions

Holding prices fixed at the pre-merger level, the post-merger margins required to generate marginal cost reductions necessary to offset upward pricing incentives are $m^1_{\text{SP}} = 47.3\%$ and $m^1_{\text{OD}} = 48.5\%$. The post-merger margins imply that Staples and Office Depot need 29.1% and 32.7% reductions in marginal costs to offset upward pricing incentives.

[28] Margins in the market for consumable office supplies to large business customers may be quite different from the overall margin reported in 10-K due to a variety of factors specific to the market, e.g., discounts and rebates. According to the companies' 10-K documents, Staples' and Office Depot's 2014 net revenues were \$5.6 billion and \$4.7 billion in the business-to-business channel.

[29] Using GUPPI to approximate merger price effects (i.e., $\ddot{p}_j \approx GUPPI_j$) à la Miller et al. (2017b) would produce conservative estimates of merger price effects.

Merger Simulation

Suppose that Staples and Office Depot are the only players in the market and that all other products are included in the outside option. The mean utilities of a representative consumer that rationalize the pre-merger market shares of Staples and Office Depot can be computed using $\log \alpha_k - \log \alpha_0 = u_k$ for $k = 1, 2$, which gives $u_{\text{SP}} = 0.807$ and $u_{\text{OD}} = 0.404$. I estimate the elasticity of the substitution parameter to be $\eta = 6.121$.[30] Solving the post-merger first-order condition gives $\ddot{p}_{\text{SP}} = 14.3\%$ and $\ddot{p}_{\text{OD}} = 18.0\%$, which turns out to be close to those of the first-order approach. The corresponding annual consumer harm, again estimated with the expression in Lemma 3 for comparison, is \$172 million.

6.5 Conclusion

Analysis with gross upward pricing pressure indices, compensating marginal cost reductions, and merger simulation all point to the same conclusion: The merger between Staples and Office Depot creates substantial merger harm, justifying the FTC's effort to prevent it.

7 Empirical Application II: Albertsons/Safeway (2015)

As a second empirical application, I consider the merger of Albertsons and Safeway, which was consummated in 2015. The FTC's approval required the divestiture of 168 stores. Quantitative evaluation of the competitive effects behind grocery mergers has been challenging due to the lack of access to or complexity associated with enormous price data. I use this empirical example to illustrate my framework's ability to analyze a large-scale retail merger in a highly tractable manner using a first-order approach.[31]

[30] Since $\epsilon^R_{jj} = \frac{1-\eta}{R_j}\frac{\partial \alpha_j}{\partial u_j}B$ and $\frac{\partial \alpha_j}{\partial u_j} = \alpha_j(1-\alpha_j)$, we have $\epsilon^R_{jj} = (1-\alpha_j)(1-\eta)$, which gives two equations to identify η. I calculate $\eta_{\text{SP}} = 6.457$ and $\eta_{\text{OD}} = 5.786$ using each equation. I take their average to arrive at $\eta = 6.121$.

[31] I do not run a merger simulation because solving the first-order conditions for thousands of stores is computationally demanding.

7.1 Background

In March 2014, AB Acquisition LLC, the parent of the Albertsons supermarket chain, agreed to terms to purchase Safeway for $9 billion.[32] Albertsons operated 1,075 stores in 28 states under the banners Albertsons, United, Amigos, and Market Street, among others. Safeway owned 1,332 stores in 18 states under the banners Safeway, Vons, Pavilions, Tom Thumb, and Randall's, among others. Upon consummation, the merger was to create the second-largest traditional grocery chain (next to Kroger) by store count and sales in the US, with approximately 2,400 stores. Figure 2 shows Albertsons and Safeway's footprint in the contiguous US in 2009.

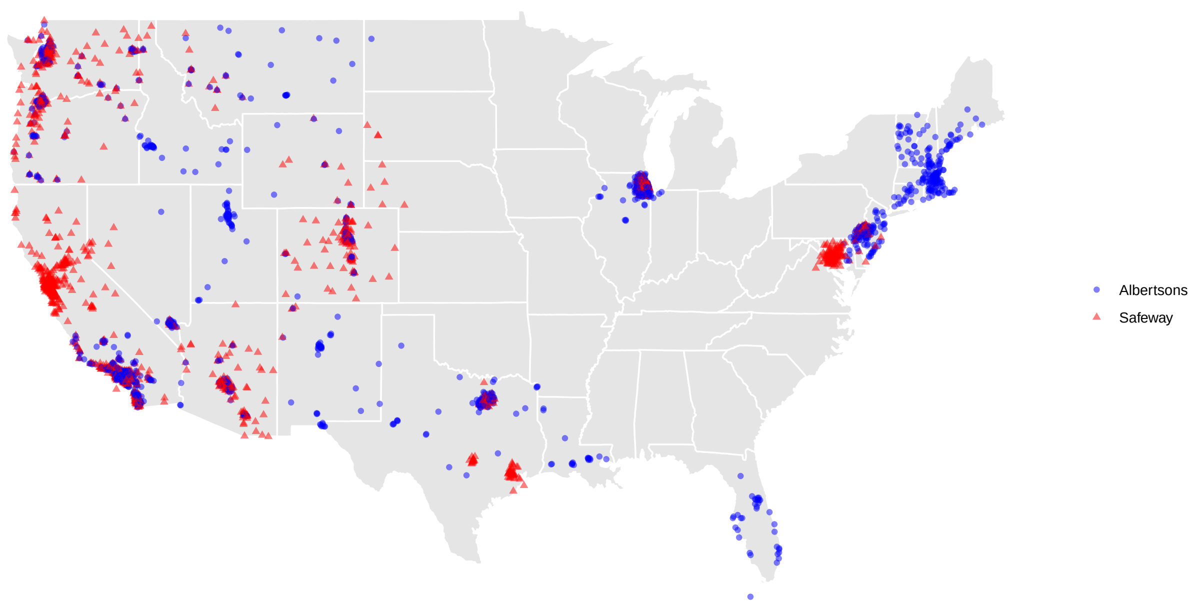


Figure 2: Albertsons and Safeway footprint in 2009

The FTC defined the relevant product market as supermarkets within "hypermarkets." Supermarkets refer to "traditional full-line retail grocery stores that sell, on a large-scale basis, food and non-food products that customers regularly consume at home—including, but not limited to, fresh meat, dairy products, frozen foods, beverages, bakery goods, dry groceries, detergents, and health and beauty products." Hypermarkets include chains such as Walmart Supercenters that sell an array of products not found in traditional supermarkets but also offer goods and services available at conventional supermarkets.

[32] I refer the reader to Federal Trade Commission (2015) for publicly available case materials that include the FTC complaint and the list of divestiture stores.

The FTC defined the relevant geographic markets as areas that range from a two- to ten-mile radius around each party's supermarkets, where the radius depends on factors such as population density, traffic, and unique market characteristics. The agency identified overlapping territories in Arizona, California, Colorado, Montana, Nevada, Oregon, Texas, Washington, and Wyoming. In late 2014, the FTC settled with the parties with a mandate to divest 168 stores in the overlap markets. Figure 3 shows the locations of the divested stores.

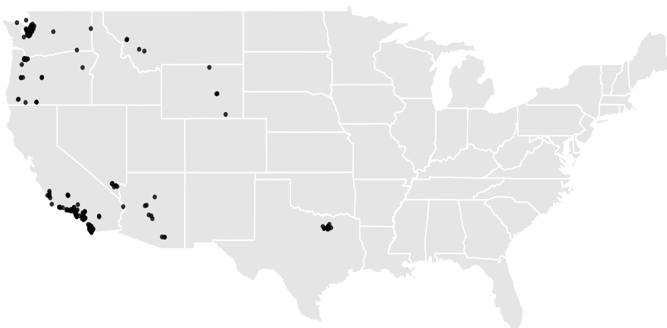


Figure 3: Location of the 168 divested stores

7.2 Data

I use the 2009 cross-section of AC Nielsen's (currently known as The Nielsen Company) Trade Dimensions TDLinx data for information on grocery stores' locations, sales, and characteristics in the US.[33] To generate conservative competitive effects estimates, I include a wide range of firms as potential competitors. Specifically, I include all grocery stores with selling space above 7,000 square feet but exclude military commissaries; my sample includes traditional supermarkets, supercenters, wholesale clubs, natural/gourmet stores, limited assortment stores, and warehouses. Since I cannot access confidential store-level margin data, I uniformly apply a relative margin of 0.27 to all stores based on the merging parties' 10-K reports around the time of the proposed merger. Finally, I obtain several census tract-level demographic variables—income, the proportion of the population with a college degree or higher, black population, and urbanicity (fraction of people

[33] Year 2009 is the closest year before 2014 for which I can access the Trade Dimensions data. In 2013, SuperValue Inc. sold Albertsons, Acme, Jewel-Osco, Shaw's and Start Market banners to Cerberus' AB Acquision, the parent company of Albertsons Inc. To capture the Albertsons/Safeway merger investigation started in 2014, I assume Albertsons owns all the banners mentioned above, which my 2009 Trade Dimensions data encodes as SuperValu-owned. Note that AC Nielsen's Trade Dimensions data estimates weekly sales volume using a proprietary algorithm.

living in census-designated urban areas)—from the IPUMS NHGIS database (Manson et al., 2023) as controls that may account for potential heterogeneity in grocery shopping preferences (Cullen et al., 2007; Charron-Chénier et al., 2017; DellaVigna and Gentzkow, 2019). Table 1 reports the summary statistics.[34]

Table 1: Summary Statistics

	Mean	St. Dev.	1st Quartile	Median	3rd Quartile
Tract Characteristics[1]					
Population (1,000)	4.716	2.191	3.287	4.436	5.784
Median Household Income ($1,000)	60.926	29.741	39.925	54.314	75.108
College	0.288	0.194	0.134	0.241	0.411
Black	0.069	0.114	0.007	0.028	0.079
Urbanicity	0.873	0.294	0.986	1.000	1.000
Store Characteristics					
Annual Revenue ($1,000,000)	18.123	18.301	6.518	11.732	22.161
Store Size (1,000 Sq. Ft.)	30.359	17.875	16.000	28.000	38.000
Supermarket	0.737	0.440	0.000	1.000	1.000
Supercenter	0.101	0.301	0.000	0.000	0.000
Wholesale Club	0.036	0.187	0.000	0.000	0.000
Natural/Gourmet	0.040	0.195	0.000	0.000	0.000
Limited Assortment	0.072	0.259	0.000	0.000	0.000
Warehouse	0.013	0.115	0.000	0.000	0.000
Big Chain	0.789	0.408	1.000	1.000	1.000
Medium Chain	0.131	0.338	0.000	0.000	0.000
Small Chain	0.080	0.271	0.000	0.000	0.000

[1] Sample consists of census tracts in overlap states.
[2] I define big chains as those with over 100 stores. Medium chains are those with 10 to 100 stores. Small chains are those with 10 or fewer stores.

7.3 Empirical Specification

I assume that each firm is an owner of multiple stores and competes by setting a uni-dimensional price at each store. Each store corresponds to a "product" in the standard Bertrand-Nash framework. Price p_j of store j is interpreted as the unobserved price index. I estimate the gross upward pricing pressure indices of the merging parties' stores before and after the merger. I do not credit merger-specific efficiency (i.e., I set $\ddot{c}_j = 0$). Thus, the GUPPI at each store j is given by

$$GUPPI_j = (1 + \epsilon_{jj}^{-1}) \sum_{k \in \mathcal{J}_B} m_k D_{j \to k}^R,$$

[34] I refer the readers to Ellickson et al. (2020) for more details on the grocery industry landscape.

where B represents the merger counterparty, and the own-price elasticity ϵ_{jj} is a known function of margins and revenue diversion ratios as shown in (7). Given store-level margin data, the estimation of revenue diversion ratios completes the GUPPI calculation.

I estimate the revenue diversion ratios using a parametric model that follows Ellickson et al. (2020). I make the following assumptions. I designate the relevant unit of consumers to be census-delineated tracts, i.e., I take each tract as a representative consumer of the population residing in the tract. Each representative consumer considers all stores within 10-mile radius, which determines the consideration sets $\mathcal{C}_i$. Each representative agent has a nested CES preference. Let the set of nests $\mathcal{B}$ include 6 nests: supermarket, supercenter, wholesale club, natural/gourmet, limited assortment, and the outside option. Consumer i's budget share spent on store j in nest b is

$$\alpha_{ij} = s_b^i s_{j|b}^i, \tag{20}$$

where

$$s_b^i = \left(\frac{\exp(\mu_b I_{i,b})}{\sum_{q \in \mathcal{B}} \exp(\mu_q I_{i,q})} \right),$$
$$s_{j|b}^i = \left(\frac{\exp(u_{ij}/\mu_b)}{\sum_{l \in \mathcal{C}_{i,b}} \exp(u_{il}/\mu_b)} \right),$$

$\mu_b \in [0,1]$ is the nesting parameter, $\mathcal{C}_{i,b}$ is the available options in nest b, and $I_{i,b} \equiv \log \sum_{l \in \mathcal{C}_{i,b}} \exp(u_{i,l}/\mu_b)$ is the inclusive value of nest b.[35] The outside good forms a distinct nest, and its nesting parameter is normalized to $\mu_0 = 1$. Given that Ellickson et al. (2020)'s results show that the nesting parameters have similar values, I simplify the estimation problem by assuming that $\mu_b = \mu$ for $b \in \mathcal{B} \backslash \{0\}$. The latent utility indices can be projected to the set of observable characteristics as

$$u_{ij} = x_{ij}^\top \theta. \tag{21}$$

The covariate vector x_{ij} includes distance from tract to store (in miles), tract characteristics, and store characteristics. I follow Ellickson et al. (2020) and assume that the set of controls can absorb the unobserved price terms, but I use a more parsimonious set of controls than Ellickson et al.

[35] If $\mu_b \to 1$ for all nests, the model collapses to the standard logit case. Consumers substitute only within each nest if $\mu_b \to 0$ for all nests.

(2020) for tractability. Finally, I assume each tract's grocery budget is 13% of its total income based on Ellickson et al. (2020)'s estimates.[36] I estimate the utility parameter θ by minimizing the distance between the observed store revenues and the model-implied store revenues via nonlinear least squares.

7.4 Estimation Results

Utility Parameters

Table 2 reports the utility parameter estimates from the nonlinear least squares problem. All coefficients have expected signs. For example, on average, consumers dislike traveling far, especially more when they are in an urban area. The coefficients on urbanicity, income, and college indicator are negative, which would be consistent with urban, higher-income, or higher-educated residents having lower grocery expenditures on average because they can substitute more for outside options such as restaurants or food delivery. Black consumers also spend less on groceries on average.[37] Consumers spend more in larger stores and at supercenters. Consumers value major banners such as Walmart, Costco, H-E-B, Whole Foods, and Trader Joe's. Finally, the nesting parameter μ is estimated to be 0.46, which indicates consumers perceive different grocery formats as highly differentiated.[38]

Revenue Diversion Ratios

Figure 4 shows the distribution of revenue diversion ratios implied at the estimated utility parameter.[39] Figure 4a reports the distribution of revenue diversion ratios for pairs of stores within 3 miles of each other.[40] The revenue diversion ratios from one store to another tend to be small. However, Figure 4b illustrates the total revenue diversion ratio from one store to *all surrounding*

[36]The grocery budget estimate is likely to be conservative. The 2014 Bureau of Labor Statistics Consumer Expenditure Survey reports that consumers spend approximately 12.6% of their pre-tax income on food but only 7.4% on food at home, which may be more relevant for estimating the grocery budget.

[37]One explanation is that black neighborhoods have fewer supermarkets; see, e.g., Bower et al. (2014) and Charron-Chénier et al. (2017). The current framework does not capture how grocery store entries endogenously depend on neighborhood characteristics.

[38]My estimate of the nesting parameter is smaller than those found in Ellickson et al. (2020), which reports $\mu \approx 0.75$ using 2006 TDLinx data. The differences may be attributed to multiple factors such as data years, model specification, and classification of nests.

[39]I use Mansley et al. (2019)'s notes to derive the tract-level weights and diversion ratios with nested CES demand.

[40]Figures' x-axes are truncated at 0.25 for presentation.

Table 2: Utility parameter estimates

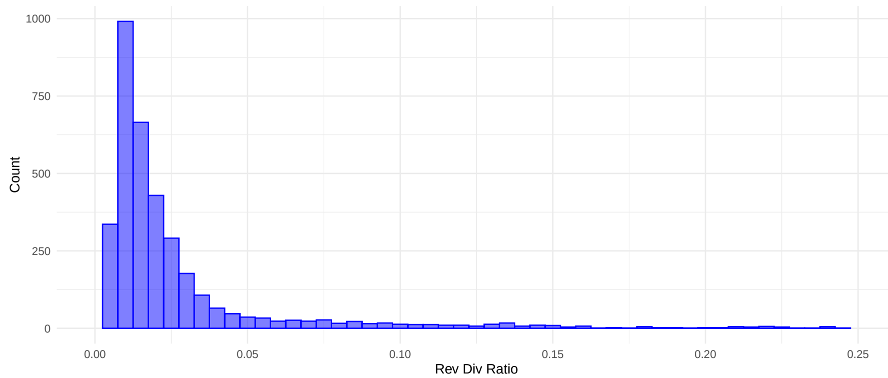
	Coef.	S.E.		Coef.	S.E.
Constant	24.99	(1.00)	Albertsons	0.23	(0.01)
Distance	−0.03	(0.01)	Safeway	0.32	(0.01)
Distance * Urbanicity	−0.05	(0.01)	Walmart	0.46	(0.02)
Urbanicity	−0.24	(0.10)	Costco	0.64	(0.04)
log(Median HH Income)	−2.57	(0.10)	Kroger	0.28	(0.01)
College	−1.71	(0.16)	H-E-B	0.52	(0.03)
Black	−4.87	(0.27)	Whole Foods	0.53	(0.03)
log(Store Size)	0.31	(0.01)	Trader Joes	0.63	(0.03)
Supermarket	0.03	(0.02)	Save Mart	0.20	(0.02)
Supercenter	0.16	(0.03)	Winco	0.44	(0.03)
Wholesale Club	−0.22	(0.04)	Stater Bros	0.47	(0.02)
Natural/Gourmet	−1.01	(0.04)	Raleys	0.29	(0.02)
Limited Assortment	−1.76	(0.05)	Target	0.08	(0.03)
Big Chain	−0.03	(0.01)	μ	0.46	(0.01)
Medium Chain	0.05	(0.01)			
Residual S.E.	0.476				

merger counterparty stores can be significantly larger, indicating the importance of accounting for multi-store ownership in retail merger analysis. GUPPI statistics enable researchers to summarize how diversions to a network of surrounding stores aggregate, which is crucial for analyzing competition in the grocery industry.

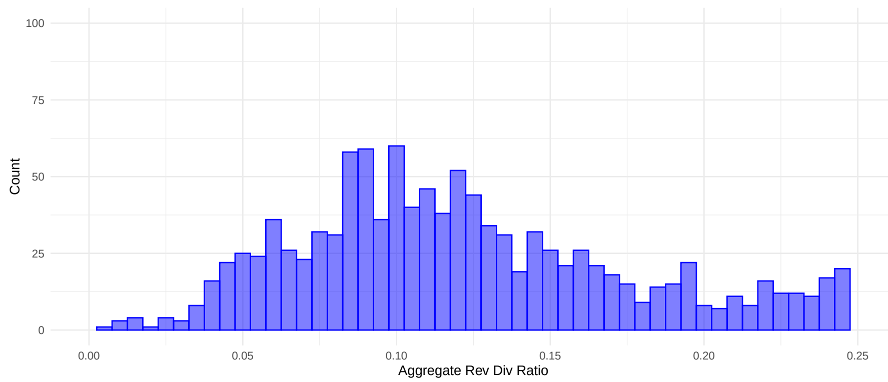
Gross Upward Pricing Pressure Indices

The FTC-mandated divestiture contributed to a substantial reduction in the GUPPIs. Table 3 reports the distribution of GUPPIs. Column "Pre" reports the store-level GUPPIs in the pre-divestiture regime. Column "Post" reports the GUPPIs of the remaining stores after the divestiture.[41] Finally, Column "Divested" reports the GUPPIs of the divested stores prior to the merger. Overall, the divestiture significantly decreased the upward pricing pressures at many problematic stores; Figure 5 shows that the divestiture induced a leftward shift in the GUPPI distribution. The divested stores had relatively high GUPPIs in proportion compared to the overall distribution reported in the first column. Post divestiture, the number of high-GUPPI stores decreases substantially.

[41] The table lists the divestiture of 165 stores, which is slightly less than the actual divestiture of 168 stores. This discrepancy of three stores arises from the time gap between my data, collected in 2009, and the year of the merger proposal in 2014.



(a) Revenue diversion ratios to stores within 3 miles



(b) Aggregate revenue diversion ratios to merger counterparty stores

Figure 4: Distribution of revenue diversion ratios

Table 3: Distribution of GUPPIs before and after the divestiture

GUPPI	Pre	Post	Divested
0–1%	430	555	0
1–2%	348	440	9
2–3%	380	229	35
3–4%	195	113	32
4–5%	117	66	28
5%–	168	63	61
Total	1,638	1,466	165

Welfare Effects of Merger

Based on Miller et al. (2017b)'s results, I approximate merger price effects as $\ddot{p}_j \approx GUPPI_j$.[42] The estimated annual consumer harm before the divestiture is \$621 million. Post-merger, the annual

[42] Although it may be possible to calculate the merger pass-through matrix using the nested CES demand assumption, I use Miller et al. (2017b)'s approach to avoid calculating a large-dimensional matrix and generate conservative estimates of merger price effects.

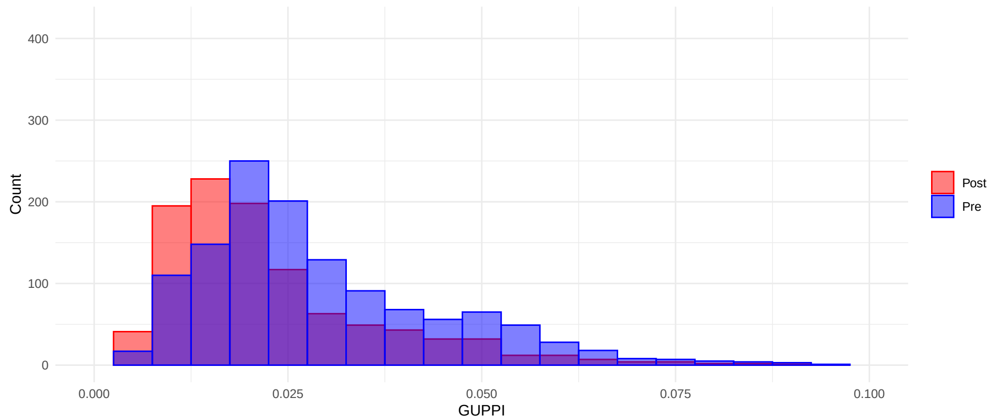


Figure 5: Distribution of GUPPIs pre and post divestiture

consumer harm reduces to $383 million.

Compensating Cost Efficiencies to Offset Upward Pricing Pressures

The consumer harm remains substantial because I assume no efficiency credit (i.e., $\ddot{c}_j = 0$). If the merger induces reductions in marginal costs, then the consumer harm will be smaller. In practice, it is common for antitrust authorities to evaluate merger effects after crediting some degree of marginal cost efficiencies. Like Nocke and Whinston (2022), I study the antitrust authorities' beliefs in firms' merger-induced marginal cost reductions by estimating the percentage change in marginal costs $\ddot{c}_j$'s that offset the upward pricing pressures.[43]

Figure 6 reports the distribution of cost-efficiency credits required to offset upward pricing pressures at the merging firms' stores before and after the divestiture. The 90th, 95th, and 99th quantiles of the post-divestiture compensating cost efficiencies are 5.2%, 6.6%, and 9.9%, respectively. If the FTC hypothetically were to credit cost efficiencies of 5%, 6%, or 7% to all stores, it would predict no price increase at more than 90%, 95%, or 99% of the remaining stores, respectively.

7.5 Limitations

My analysis has several limitations. First, the data quality used in this application is more restricted than what would be available during actual merger reviews. For example, the TDLinx data re-

[43] From (4), the net gross upward pricing pressure index is $GUPPI_j = GUPPI_j^{\text{no credit}} + \ddot{c}_j(1 - m_j)$. Thus, the percentage reduction in marginal cost to offset upward pricing pressure at store j is $-\ddot{c}_j = GUPPI_j^{\text{no credit}}/(1 - m_j)$. This measure is easier to calculate than Werden (1996)'s compensating marginal cost reductions, which account for all firms' optimal pricing equations simultaneously.

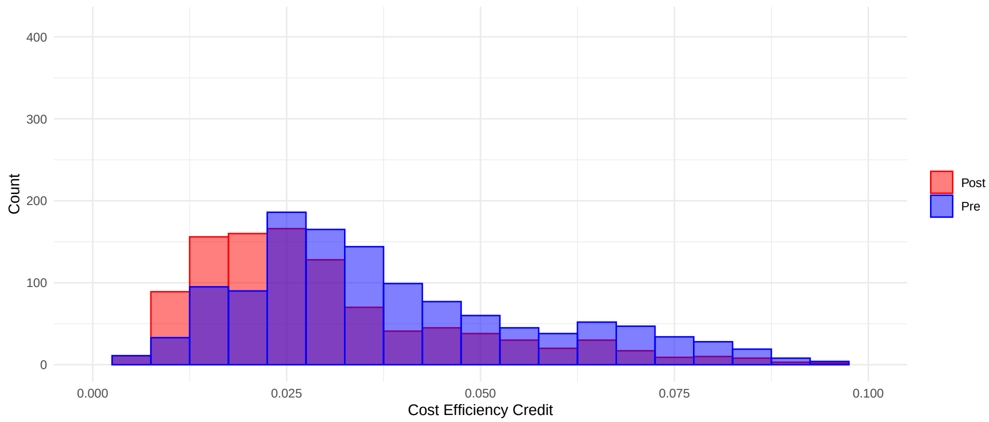


Figure 6: Distribution of compensating cost efficiencies pre and post divestiture

lies heavily on an unknown proprietary imputation method to estimate revenues. In practice, antitrust authorities can request actual store-level sales/margins data or detailed consumer-level transaction data from firms operating in the overlap markets (Hosken and Tenn, 2016). Such data would improve the quality of the revenue diversion ratio estimates and the consumer budget share parameter.

Second, my GUPPI estimates can be sensitive to modeling assumptions because diversion ratio estimates depend on the specification of the nesting structure. In practice, the contention between the antitrust authority and the merging parties may revolve around the definition of the relevant antitrust market, which also influences estimates of consumers' overall budget and nesting structures. For example, in *Whole Foods/Wild Oats*, the FTC alleged a market consisting of "premium, natural, and organic supermarkets," which would result in a substantially narrower market than what I use for the current application. Estimating a flexible random coefficient model can improve the quality of the estimates with weaker assumptions but can be challenging without price data.

8 Conclusion

Throughout this paper, I have illustrated that a lack of price data may not be a barrier to conducting a thorough unilateral effects analysis for horizontal mergers. Data on revenues, profit margins, and revenue diversion ratios can be sufficient to identify the price and welfare implications of mergers. When supplemented with additional assumptions about consumer demand, revenue diversion ratios become identifiable from cross-sectional data on consumers' expenditures, and merger simulations are feasible. The approach detailed in this study offers broad applicability to various industries in scenarios where access to price data is limited.

I see several avenues for future research. First, finding alternative methods for estimating the key statistics used in my framework—diversion ratios, margins, and pass-through rates—will be interesting as these statistics are not always easy to obtain. Econometric approaches that can relax the homogeneous price responsiveness assumption to estimate diversion ratios will be helpful. Furthermore, developing alternative methods to obtain reliable pass-through rates will increase the attractiveness of the first-order approach to merger analysis.

Second, developing empirical and simulation results under various data scenarios will be useful. In many settings, the researcher can access partial price and margin information. For example, observed prices and margins may be aggregated across multiple products. While I have assumed away these scenarios to focus on identification arguments, understanding how to leverage partial information on prices and margins and how aggregation error can affect the results appear to be important questions for empirical researchers.

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Appendix

A Proofs

A.1 Proof of Lemma 1

A.1.1 Proof of Lemma 1.1

Proof. Recall $\epsilon_{jj} = \frac{\partial q_j}{\partial p_j}\frac{p_j}{q_j}$, $\epsilon_{kj} = \frac{\partial q_k}{\partial p_j}\frac{p_j}{q_k}$, $\epsilon^R_{jj} = \frac{\partial R_j}{\partial p_j}\frac{p_j}{R_j}$, $\epsilon^R_{kj} = \frac{\partial R_k}{\partial p_j}\frac{p_j}{R_k}$. Using these objects, it is straightforward to verify $D_{j\to k} = -\frac{\epsilon_{kj}}{\epsilon_{jj}}\frac{q_k}{q_j}$ and $D^R_{j\to k} = -\frac{\epsilon^R_{kj}}{\epsilon^R_{jj}}\frac{R_k}{R_j}$. □

A.1.2 Proof of Lemma 1.2

Proof. From $R_k = p_k q_k$, $\frac{\partial R_k}{\partial p_j} = q_j \mathbb{I}_{j=k} + p_k\frac{\partial q_k}{\partial p_j}$ for arbitrary j and k. Then for arbitrary j and k,

$$\epsilon^R_{kj} = \frac{\partial R_k}{\partial p_j}\frac{p_j}{R_k} = \mathbb{I}_{j=k} q_j \frac{p_j}{R_k} + p_k \frac{\partial q_k}{\partial p_j}\frac{p_j}{R_k} = \mathbb{I}_{j=k} + \frac{\partial q_k}{\partial p_j}\frac{p_j}{q_k} = \mathbb{I}_{j=k} + \epsilon_{kj}.$$

□

A.1.3 Proof of Lemma 1.3

Proof. Finally, by definition, $R_k = p_k q_k$, so $\frac{\partial R_k}{\partial p_j} = \mathbb{I}_{j=k} q_k + p_k \frac{\partial q_k}{\partial p_j}$ for arbitrary j and k. Then

$$\begin{aligned} D^R_{j\to k} &= -\frac{\mathbb{I}_{j=k} q_k + p_k \frac{\partial q_k}{\partial p_j}}{q_j + p_j \frac{\partial q_j}{\partial p_j}} \\ &= -\frac{\mathbb{I}_{j=k}(\frac{q_k}{p_j})/(\frac{\partial q_k}{\partial p_j}) + \frac{p_k}{p_j}(\frac{\partial q_k}{\partial p_j})/(\frac{\partial q_j}{\partial p_j})}{(\frac{q_j}{p_j})/(\frac{\partial q_j}{\partial p_j}) + 1} \\ &= -\frac{\mathbb{I}_{j=k}\epsilon^{-1}_{kj} - (\frac{p_k}{p_j})D_{j\to k}}{\epsilon^{-1}_{jj} + 1}. \end{aligned}$$

Rewriting the last line gives $(1+\epsilon^{-1}_{jj})D^R_{j\to k} + \mathbb{I}_{j=k}\epsilon^{-1}_{kj} = D_{j\to k}\frac{p_k}{p_j}$. But as $\mathbb{I}_{j=k}\epsilon^{-1}_{kj} = \mathbb{I}_{j=k}\epsilon^{-1}_{jj}$, we have $(1+\epsilon^{-1}_{jj})D^R_{j\to k} + \mathbb{I}_{j=k}\epsilon^{-1}_{jj} = D_{j\to k}\frac{p_k}{p_j}$. □

A.2 Proof of Lemma 2

Proof. Substituting in $(1+\epsilon_{jj}^{-1})D_{j\to k}^{R} = D_{j\to k}\frac{p_k}{p_j}$ to firm F's first-order condition with respect to p_j gives

$$m_j = -\epsilon_{jj}^{-1} + \sum_{k\in\mathcal{J}_F\setminus j} m_k D_{j\to k}\frac{p_k}{p_j} = -\epsilon_{jj}^{-1} + (1+\epsilon_{jj}^{-1})\sum_{k\in\mathcal{J}_F\setminus j} m_k D_{j\to k}^{R}.$$

Solving for ϵ_{jj} gives the desired expression (7). □

A.3 Proof of Proposition 1

Proof. The statement follows because cost efficiencies $\ddot{c}_j$, margins m_j, revenue diversion ratios $D_{j\to k}^{R}$ are observed, and the own-price elasticity ϵ_{jj} is a known function of margins and revenue diversion ratios. □

A.4 Proof of Proposition 2

Proof. The statement follows because the first three assumptions ensure the identification of GUPPIs, and the last assumption ensures that GUPPIs can be translated to first-order approximation of merger price effects. □

A.5 Proof of Lemma 3

Proof. The changes in consumer surplus and the producer surplus from a price change are

$$\Delta CS_j \approx -\frac{1}{2}(p_j^1 - p_j^0)(q_j^1 + q_j^0)$$
$$\Delta PS_j \approx \Delta p_j \times q_j^1 + \Delta q_j \times (p_j^0 - c_j^0) - \Delta c_j \times q_j^1.$$

As $q_j^1 \approx q_j^0 + (\partial q_j/\partial p_j)(p_j^1 - p_j^0) = q_j^0(1+\epsilon_{jj}\ddot{p}_j)$, $\Delta q_j \equiv q_j^1 - q_j^0 \approx \epsilon_{jj} q_j^0 \ddot{p}_j$. Moreover, $-\Delta c_j/p_j^0 = -\ddot{c}_j c_j^0/p_j^0 = -\ddot{c}_j(1-m_j^0)$. Then

$$\Delta CS_j \approx -\frac{1}{2}\Delta p_j q_j^0(2+\epsilon_{jj}\ddot{p}_j) = -\ddot{p}_j R_j^0(1+\frac{1}{2}\epsilon_{jj}\ddot{p}_j).$$

Next,

$$\Delta PS_j \approx \ddot{p}_j R_j^0(1+\epsilon_{jj}\ddot{p}_j) + \epsilon_{jj} R_j^0 \ddot{p}_j m_j^0 - \frac{\Delta c_j}{p_j} R_j^0 (1+\epsilon_{jj}\ddot{p}_j).$$

But $\Delta c_j/p_j = \frac{\Delta c_j}{c_j}\frac{c_j}{p_j} = \ddot{c}_j(1-m_j)$. □

A.6 Proof of Proposition 3

Proof. The assumptions ensure that the own-price elasticities of demand and merger price effects are identified. Combining them with data on revenues, margins, and cost efficiencies, ΔCS_j's and ΔPS_j's are identified. □

A.7 Proof of Lemma 4

Proof. Suppose Assumption 5 holds. In Online Appendix A, I show that the compensating marginal cost reductions are defined by equations (22) and (23). Replacing the terms $D_{j\to l}\frac{p_l}{p_j}$ with $(1+\epsilon_{jj}^{-1})D^R_{j\to l}$ (Lemma 1) yields (11). □

A.8 Proof of Proposition 4

Proof. Using the fact that $(1+\epsilon_{jj}^{-1})D^R_{j\to k} = \mathbb{I}_{j=k}\epsilon_{kj}^{-1} = D_{j\to k}\frac{p_k}{p_j}$, the post-merger first order conditions can be rewritten as

$$-\epsilon_{jj}^{-1} - m_j^1 + (1+\epsilon_{jj}^{-1})\sum_{l\in\mathcal{J}_A\setminus j} m_l^1 D^R_{j\to k} + (1+\epsilon_{jj}^{-1})\sum_{k\in\mathcal{J}_B} m_k^1 D^R_{j\to k} = 0.$$

As the own-price elasticities and revenue diversion ratios are identified, the post-merger margins are identified from the post-merger first-order conditions. □

A.9 Proof of Lemma 5

Proof. The revenue diversion ratio from product j to k, assuming that Leibniz's rule applies, is

$$\begin{aligned} D^R_{jk} &= -\frac{\int_{i\in\mathcal{S}_k}\partial e_{ik}/\partial p_j d\mu}{\int_{\tilde{i}\in\mathcal{S}_j}\partial e_{\tilde{i}j}/\partial p_j d\mu} \\ &= -\int_{i\in\mathcal{S}_k}\left(\frac{\partial e_{ik}/\partial p_j}{\int_{i\in\mathcal{S}_j}\partial e_{ij}/\partial p_j d\mu}\right)d\mu \\ &= \int_{i\in\mathcal{S}_k}\left(\frac{\partial e_{ij}/\partial p_j}{\int_{i\in\mathcal{S}_j}\partial e_{ij}/\partial p_j d\mu}\right)\left(-\frac{\partial e_{ik}/\partial p_j}{\partial e_{ij}/\partial p_j}\right)d\mu, \end{aligned}$$

which is what I wanted to show. □

A.10 Proof of Lemma 6

Proof. Under Assumption 6, I have $\frac{\partial e_{ik}}{\partial p_j} = \frac{\partial e_{ik}}{\partial u_{ij}} \frac{\partial u_{ij}}{\partial p_j}$. As $\frac{\partial u_{ik}}{\partial p_j}$ is independent of i, it cancels out in the numerators and the denominators of w_{ij} and $D^R_{i,j \to k}$, yielding the desired expressions. □

A.11 Proof of Proposition 5

Proof. Omitted. □

A.12 Proof of Lemma 7

Proof. I omit the derivation of the multinomial logit functional form for α_{ij} as it is standard. Under Assumption 8, as $\frac{\partial \alpha_{ik}}{\partial u_{ij}} = \alpha_{ik}(\mathbb{I}_{j=k} - \alpha_{ij})\mathbb{I}_{j,k \in \mathcal{C}_i}$, I can simplify the expression to (15) with

$$w_{ij} = \frac{\alpha_{ij}(1-\alpha_{ij})B_i}{\sum_{\tilde{i} \in \mathcal{S}_j} \alpha_{\tilde{i}j}(1-\alpha_{\tilde{i}j})B_{\tilde{i}}},$$

$$D^R_{ijk} = \frac{\alpha_{ik}}{1-\alpha_{ij}},$$

which is what I wanted to show. □

A.13 Proof of Proposition 6

Proof. Omitted. □

A.14 Proof of Proposition 7

Proof. Let $v_{ij} \equiv \exp(u_{ij})$. The revenue of product k before the removal of product j is $R_k^{\text{pre}} = \int_{i \in \mathcal{S}_k} \alpha_{ik}^{\text{pre}} B_i d\mu$ where $\alpha_{ij}^{\text{pre}} = \frac{v_{ik}}{\sum_{l \in \mathcal{C}_i} v_{il}}$. After removing product j, the revenue of product k becomes $R_k^{\text{post}} = \int_{i \in \mathcal{S}_k} \alpha_{ik}^{\text{post}} B_i d\mu$ with $\alpha_{ik}^{\text{post}} = \frac{v_{ik}}{\sum_{l \in \mathcal{C}_i \setminus j} v_{il}}$. The revenue diversion ratio associated with a removal of product j is

$$
\begin{aligned}
D_{jk}^{\text{2nd}} &= -\frac{R_k^{\text{post}} - R_k^{\text{pre}}}{0 - R_j^{\text{pre}}} \\
&= \frac{\int_{i \in \mathcal{S}_k} (\alpha_{ik}^{\text{post}} - \alpha_{ik}^{\text{pre}}) B_i d\mu}{\int_{i \in \mathcal{S}_j} \alpha_{ij}^{\text{pre}} B_i d\mu} \\
&= \int_{i \in \mathcal{S}_k} \left(\frac{\alpha_{ij}^{\text{pre}} B_i}{\int_{i \in \mathcal{S}_j} \alpha_{ij}^{\text{pre}} B_i d\mu} \right) \left(\frac{\alpha_{ik}^{\text{post}} - \alpha_{ik}^{\text{pre}}}{\alpha_{ij}^{\text{pre}}} \right) d\mu.
\end{aligned}
$$

Thus, it is sufficient to verify that $\frac{\alpha_{ik}^{\text{post}} - \alpha_{ik}^{\text{pre}}}{\alpha_{ij}^{\text{pre}}} = \frac{\alpha_{ik}^{\text{pre}}}{1 - \alpha_{ij}^{\text{pre}}}$. Observe

$$
\begin{aligned}
\frac{\alpha_{ik}^{\text{post}} - \alpha_{ik}^{\text{pre}}}{\alpha_{ij}^{\text{pre}}} &= \frac{\frac{v_{ik}}{\sum_{l \in \mathcal{C}_i \setminus j} v_{il}} - \frac{v_{ik}}{\sum_{l \in \mathcal{C}_i} v_{il}}}{\frac{v_{ij}}{\sum_{l \in \mathcal{C}_i} v_{il}}} \\
&= \frac{\left(\frac{\sum_{l \in \mathcal{C}_i} v_{il}}{\sum_{l \in \mathcal{C}_i \setminus j} v_{il}} \right) v_{ik} - v_{ik}}{v_{ij}} \\
&= \frac{v_{ik}}{v_{ij}} \left(\frac{v_{ij}}{\sum_{l \in \mathcal{C}_i \setminus j} v_{il}} \right) \\
&= \frac{\frac{v_{ik}}{\sum_{l \in \mathcal{C}_i} v_{il}}}{1 - \frac{v_{ij}}{\sum_{l \in \mathcal{C}_i} v_{il}}},
\end{aligned}
$$

which is the desired expression. □

A.15 Proof of Lemma 8

Proof. Given that the latent utility function is specified as $u_{ij} = \log \beta_{ij} + (1 - \eta) \log p_j$, as $p_j^{\text{post}} \equiv p_j^{\text{pre}}(1 + \ddot{p}_j)$, the pre- and post-merger latent utilities are related as $u_{ij}^{\text{post}} = u_{ij}^{\text{pre}} + (1 - \eta) \log(1 + \ddot{p}_j)$. Thus, if u_{ij}^{pre} and η are identified, u_{ij}^{post} is a known function of $\ddot{p}_j$. It follows that for arbitrary $\ddot{p}$, u_{ij}^{post}, $\alpha_{ij}^{\text{post}}$, e_{ij}^{post}, R_j^{post} can be computed for all i and j. Furthermore, as

$$
\begin{aligned}
\epsilon_{jj}^R &= \frac{1 - \eta}{R_j} \sum_{i \in \mathcal{S}_j} \frac{\partial \alpha_{ij}}{\partial u_{ij}} B_i, \\
D_{j \to l}^R &= -\frac{\sum_{i \in \mathcal{S}_l} \frac{\partial \alpha_{il}}{\partial u_{ij}} B_i}{\sum_{i \in \mathcal{S}_j} \frac{\partial \alpha_{ij}}{\partial u_{ij}} B_i},
\end{aligned}
$$

$\epsilon_{jj}^{R,\text{post}}$ and $D_{j\to l}^{R,\text{post}}$ can be computed for arbitrary j and l. Finally, assuming that the marginal cost does not change,

$$m_j^{\text{post}} = \frac{p_j^{\text{post}} - c_j^{\text{pre}}}{p_j^{\text{post}}} = 1 - \frac{1}{1 + \ddot{p}_j}(1 - m_j^{\text{pre}}).$$

Thus, each m_j^{post} is also a known function of $\ddot{p}_j$. □

A.16 Proof of Proposition 8

Proof. Omitted. □

Online Appendix for

Merger Analysis with Latent Price

Paul S. Koh

February 5, 2025

A Review of Unilateral Effects Analysis

In this section, I review the derivation of firms' profit maximization conditions, gross upward pricing pressure indices, and compensating marginal cost reductions.

A.1 Firm's Problem

The multiproduct firms engage in a Bertrand-Nash pricing game. Each firm $F \in \mathcal{F}$ maximizes its total profit $\sum_{j\in\mathcal{J}_F} \pi_j$ with respect to a vector of prices $(p_j)_{j\in\mathcal{J}_F}$. The first-order condition with respect to p_j is

$$q_j + (p_j - c_j)\frac{\partial q_j}{\partial p_j} + \sum_{l\in\mathcal{J}_F\setminus j}(p_l - c_l)\frac{\partial q_l}{\partial p_j} = 0.$$

Normalizing the above to be quasilinear in the marginal cost gives

$$-p_j\epsilon_{jj}^{-1} - (p_j - c_j) + \sum_{l\in\mathcal{J}_F\setminus j}(p_l - c_l)D_{j\to l} = 0,$$

where $\epsilon_{jj} \equiv \frac{\partial q_j}{\partial p_j}\frac{p_j}{q_j}$ is the own-price elasticity of demand, and $D_{j\to l} \equiv -\frac{\partial q_l/\partial p_j}{\partial q_j/\partial p_j}$ is the quantity diversion ratio from product j to product l.[44] Further dividing the FOC by p_j normalizes the first-order conditions to be quasilinear in margins and gives (1):

$$-\epsilon_{jj}^{-1} - m_j + \sum_{l\in\mathcal{J}_F\setminus j} m_l D_{j\to l}\frac{p_l}{p_j} = 0.$$

[44] Rearranging the first-order conditions gives the optimal markup equation $p_j(1+\epsilon_{jj}^{-1}) = c_j + \sum_{l\in\mathcal{J}_F\setminus j}(p_l - c_l)D_{j\to l}$, which in turn implies that $(1+\epsilon_{jj}^{-1}) > 0$, or, equivalently, $\varepsilon_{jj} < -1$, i.e., in a Bertrand-Nash equilibrium, firms always price at the elastic region of demand.

A.2 Upward Pricing Pressure

Consider a merger between two firms A and B. The upward pricing pressure associated with product j of firm A is defined as

$$UPP_j \equiv \Delta c_j + \sum_{k \in \mathcal{J}_B} (p_k - c_k) D_{j \to k},$$

where $\Delta c_j = c_j^{\text{post}} - c_j^{\text{pre}} < 0$ represents the reduction in marginal cost due to merger-specific efficiencies; those of firm B are defined symmetrically.[45] The gross upward pricing pressure index is obtained by normalizing upward pricing pressure by price, i.e., $GUPPI_j \equiv UPP_j / p_j$ and can be rewritten as

$$GUPPI_j = \ddot{c}_j (1 - m_j) + \sum_{k \in \mathcal{J}_B} m_k D_{j \to k} \frac{p_k}{p_j},$$

where $\ddot{c}_j \equiv \Delta c_j / c_j^{\text{pre}} \in (-1, 0)$ represents the percentage decrease in marginal cost from the pre-merger equilibrium.

A.3 Compensating Marginal Cost Reduction

Compensating marginal cost reductions (CMCR) are defined as the percentage decrease in the merging parties' costs that would leave the pre-merger prices unchanged after the merger. Werden (1996) shows that CMCRs are identified when prices, margins, and diversion ratios are observed.[46] Specifically, under Assumption 5, the compensating marginal cost reductions (defined in percentage change relative to pre-merger levels) are

$$\ddot{c}_j = \frac{m_j^0 - m_j^1}{1 - m_j^0}. \tag{22}$$

The post-merger margins are defined by the post-merger first-order conditions

$$-\epsilon_{jj}^{-1} - m_j^1 + \sum_{l \in \mathcal{J}_A \backslash j} m_l^1 D_{j \to l} \frac{p_l}{p_j} + \sum_{k \in \mathcal{J}_B} m_k^1 D_{j \to k} \frac{p_k}{p_j} = 0. \tag{23}$$

Finding post-merger margins amounts to solve a linear system of equations.

[45] I occasionally omit the superscript "pre" when it is clear the object is that of the pre-merger equilibrium.
[46] Werden (1996) derives the expressions for a single-product firms case.

To see that (22) holds, note that, since $p_j^1 = p_j^0$,

$$m_j^0 - m_j^1 = \frac{1}{p_j^0}((p_j^0 - c_j^0) - (p_j^0 - c_j^1)) = \frac{\Delta c_j}{p_j^0}.$$

Then

$$\frac{m_j^0 - m_j^1}{1 - m_j^0} = \frac{\frac{\Delta c_j}{p_j}}{\frac{p_j^0}{p_j^0} - \frac{p_j^0 - c_j^0}{p_j^0}} = \frac{\Delta c_j / p_j^0}{c_j^0 / p_j^0} = \frac{\Delta c_j}{c_j^0} = \ddot{c}_j.$$

B Identification of Compensating Variation Under CES Preferences

It is possible to show that the compensating variation associated with a price increase is identified under the CES utility assumption. In the previous sections, I showed that consumer surplus variations can be approximated. When the utility function takes a CES form, it is possible to derive an exact formula for compensating variation associated with a vector of simultaneous price changes.

Let $V_i(p, B_i)$ be consumer i's indirect utility at price p and budget B_i. The compensating variation for consumer i associated with a price increase from p^0 to p^1 is defined as $CV_i \in \mathbb{R}$ that solves $V_i(p^0, B_i) = V_i(p^1, B_i - CV_i)$. The total compensating variation is $CV = \int_{i \in \mathcal{I}} CV_i d\mu$. Under the CES utility assumption, the compensating variation admits a closed-form expression.

Lemma 9 (Compensating variation under CES utility)**.** *Suppose Assumption 8 holds. Let* $P_i(u_i) \equiv (\sum_{k \in \mathcal{C}_i} \exp(u_{ik}))^{\frac{1}{\eta - 1}}$. *For each consumer i, the compensating variation associated with a price change from p^0 to p^1 is*

$$CV_i = B_i \left(1 - \frac{P_i(u_i^0)}{P_i(u_i^1)}\right) \tag{24}$$

where $u_i^0 = u_i(p^0)$ *and* $u_i^1 = u_i(p^1)$. *Furthermore,* $u_{ij}^1 = u_{ij}^0 + (1 - \eta)\log(1 + \ddot{p}_j)$.

Proof. Consumer i's indirect utility function is

$$V_i(p, B_i) = A P_i^{\beta,\eta}(p) B_i \tag{25}$$

where $P_i^{\beta,\eta}(p) \equiv \left(\sum_{k \in \mathcal{C}_i} \beta_{ik} p_k^{1-\eta}\right)^{\frac{1}{\eta - 1}}$. Solving for CV_i gives the desired expression. □

Given a vector of percentage price increases $\ddot{p}$, the analyst can compute the compensating variation of each consumer if the analyst observes the consumer's budget and pre-merger mean utilities, and the price coefficient.

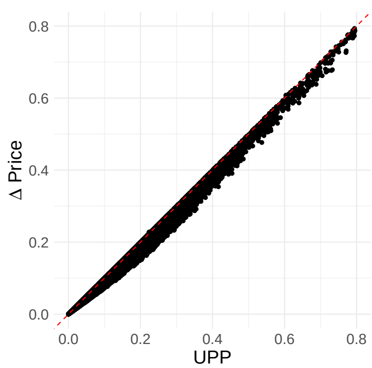
Proposition 9 (Identification of compensating variation)**.** *Suppose Assumptions 6, 7 and 8 hold. Then the compensating variation of each consumer associated with the the merger price effects is identified.*

C Accuracy of Upward Pricing Pressure

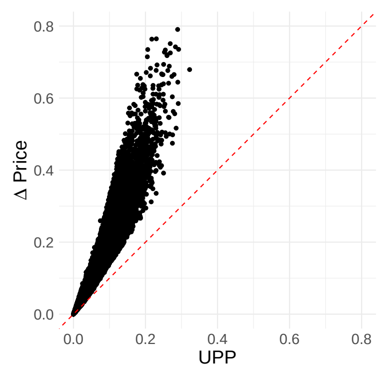
To predict merger price effects using upward pricing pressure, I have assumed that

$$\Delta p_j \approx UPP_j. \tag{26}$$

In a companion paper (Koh, 2024), I repeat the simulation exercise in Miller et al. (2017b) and examine whether (26) is reasonable. Miller et al. (2017b) does not study the case of CES demand.



(a) Logit



(b) CES

Figure 7: Accuracy of UPP

I report the simulation results in Figure 7. First, Figure 7-(a) shows that upward pricing pressure accurately predicts merger price effects in the case of logit demand. The result is also consistent with Miller et al. (2017b) (see their Figure 2). Next, Figure 7-(b) shows the simulation results with CES demand. It shows that upward pricing pressure underpredicts merger price effects. Thus, using upward pricing pressures as proxies for merger price effects would yield conservative predictions.

D Pass-Through Matrix Under CES Preference

In this section, I derive the merger pass-through matrix assuming CES preference to illustrate that the analyst can estimate pass-through rates using the merging firms' data used for GUPPI calculation. For simplicity, I assume a single representative consumer and single-product firms. I apply the results to calculate the merger pass-through matrix for the Staples/Office Depot example.

Definition of Merger Pass-through Matrix

Consider a merger between firms j and k. Let

$$h_j(\tilde{p}) \equiv -\varepsilon_{jj}^{-1} - m_j + (1 + \varepsilon_{jj}^{-1}) m_k D_{j\to k}^R,$$
$$h_k(\tilde{p}) \equiv -\varepsilon_{kk}^{-1} - m_k + (1 + \varepsilon_{kk}^{-1}) m_j D_{k\to j}^R,$$

where $\tilde{p}_j \equiv \log p_j$ for each $j \in \mathcal{J}$ so that $h(\tilde{p}^{\text{post}}) = 0$ describe the merging firms' post-merger first-order conditions. The above first-order conditions are normalized to be quasilinear in margins and expressed as functions of log prices so that we can calculate how GUPPIs translate to merger price effects in percentage change terms.

A merger pass-through matrix describes the marginal effects of (normalized) tax rates on merged firms' prices at the pre-merger equilibrium. To formalize the definition, first evaluate $h(\tilde{p})$ at the pre-merger prices to get $h(\tilde{p}^{\text{pre}}) = c$ for some vector of constants c. Introduce tax rates $\tilde{t}$ and assume that $\tilde{p} = \tilde{p}(\tilde{t})$ such that $h(\tilde{p}) + \tilde{t} = c$ around $\tilde{p}^{\text{pre}}$. By the implicit function theorem,

$$\frac{\partial \tilde{p}}{\partial \tilde{t}} \cdot \frac{\partial h(\tilde{p})}{\partial \tilde{p}} + I = 0.$$

Rearranging the above gives the merger pass-through matrix defined as

$$M \equiv \left.\frac{\partial \tilde{p}}{\partial \tilde{t}}\right|_{\tilde{t}=0} = -\left(\frac{\partial h(\tilde{p})}{\partial \tilde{p}}\right)^{-1}\Bigg|_{\tilde{p}=\tilde{p}^{\text{pre}}}$$

Then, the first-order approximation of merger price effects is

$$\ddot{p} \approx M \cdot GUPPI.$$

Derivation

Recall that under CES preference, $\varepsilon_{jj}^R = (1-\alpha_j)(1-\eta)$ and $D_{j\to k}^R = \frac{\alpha_k}{1-\alpha_j}$. Then since $\varepsilon_{jj} = \varepsilon_{jj}^R - 1$, we have $\varepsilon_{jj} = (1-\alpha_j)(1-\eta) - 1$. In addition, $\partial\alpha_j/\partial u_j = \alpha_j(1-\alpha_j)$, and $\partial\alpha_j/\partial u_k = -\alpha_k\alpha_j$ if $j \neq k$. Moreover, $\partial u_j/\partial\tilde{p}_j = (1-\eta)$ for all product $j \in \mathcal{J}$.

Let us derive the 2×2 merger pass-through matrix M given by

$$M = -\begin{bmatrix} \frac{\partial h_j}{\partial \tilde{p}_j} & \frac{\partial h_j}{\partial \tilde{p}_k} \\ \frac{\partial h_k}{\partial \tilde{p}_j} & \frac{\partial h_k}{\partial \tilde{p}_k} \end{bmatrix}^{-1}.$$

The derivatives of h_j with respect to $\tilde{p}_j$ and $\tilde{p}_k$ are given by

$$\frac{\partial h_j}{\partial \tilde{p}_j} = -\frac{\partial \varepsilon_{jj}^{-1}}{\partial \tilde{p}_j} - \frac{\partial m_j}{\partial \tilde{p}_j} + \frac{\partial(1+\varepsilon_{jj}^{-1})}{\partial \tilde{p}_j} m_k D_{j\to k}^R + (1+\varepsilon_{jj}^{-1}) m_k \frac{\partial D_{j\to k}^R}{\partial \tilde{p}_j}$$
$$\frac{\partial h_j}{\partial \tilde{p}_k} = -\frac{\partial \varepsilon_{jj}^{-1}}{\partial \tilde{p}_k} + \frac{\partial(1+\varepsilon_{jj}^{-1})}{\partial \tilde{p}_k} m_k D_{j\to k}^R + (1+\varepsilon_{jj}^{-1}) \frac{\partial m_k}{\partial \tilde{p}_k} D_{j\to k}^R + (1+\varepsilon_{jj}^{-1}) m_k \frac{\partial D_{j\to k}^R}{\partial \tilde{p}_k}.$$

The partial derivatives $\frac{\partial h_k}{\partial \tilde{p}_j}$ and $\frac{\partial h_k}{\partial \tilde{p}_k}$ are obtained symmetrically.

First, since $\frac{\partial \varepsilon_{jj}}{\partial \tilde{p}_j} = (-\frac{\partial \alpha_j}{\partial u_j}\frac{\partial u_j}{\partial \tilde{p}_j})(1-\eta)$ and $\frac{\partial \varepsilon_{jj}}{\partial \tilde{p}_k} = (-\frac{\partial \alpha_j}{\partial u_k}\frac{\partial u_k}{\partial \tilde{p}_k})(1-\eta)$,

$$\frac{\partial \varepsilon_{jj}}{\partial \tilde{p}_j} = (-1)\alpha_j(1-\alpha_j)(1-\eta)(1-\eta),$$
$$\frac{\partial \varepsilon_{jj}}{\partial \tilde{p}_k} = \alpha_j\alpha_k(1-\eta)(1-\eta).$$

Second, since $\frac{\partial m_j}{\partial \tilde{p}_j} = \frac{\partial(1-c_j/\exp(\tilde{p}_j))}{\partial \tilde{p}_j} = c_j/\exp(\tilde{p}_j) = c_j/p_j$,

$$\frac{\partial m_j}{\partial \tilde{p}_j} = 1 - m_j.$$

Third, from $D^R_{j\to k} = \frac{\alpha_k}{1-\alpha_j}$, we have

$$\begin{aligned}
\frac{\partial D^R_{j\to k}}{\partial \tilde{p}_j} &= \frac{\partial \alpha_k}{\partial u_j}\frac{\partial u_j}{\partial \tilde{p}_j}\frac{1}{1-\alpha_j} + \alpha_k(-1)(1-\alpha_j)^{-2}(-1)\frac{\partial \alpha_j}{\partial u_j}\frac{\partial u_j}{\partial \tilde{p}_j} \\
&= -\alpha_k\alpha_j(1-\eta)\frac{1}{1-\alpha_j} + \frac{\alpha_k}{(1-\alpha_j)^2}\alpha_j(1-\alpha_j)(1-\eta) \\
&= -\alpha_j(1-\eta)D^R_{j\to k} + \alpha_j(1-\eta)D^R_{j\to k} \\
&= 0.
\end{aligned}$$

Next,

$$\begin{aligned}
\frac{\partial D^R_{j\to k}}{\partial \tilde{p}_k} &= \frac{\partial \alpha_k}{\partial u_k}\frac{\partial u_k}{\partial \tilde{p}_k}\frac{1}{1-\alpha_j} + \alpha_k(-1)(1-\alpha_j)^{-2}(-1)\frac{\partial \alpha_j}{\partial u_k}\frac{\partial u_k}{\partial \tilde{p}_k} \\
&= \alpha_k(1-\alpha_k)(1-\eta)\frac{1}{1-\alpha_j} + \frac{\alpha_k}{(1-\alpha_j)^2}(-\alpha_j\alpha_k)(1-\eta) \\
&= (1-\alpha_k)(1-\eta)D^R_{j\to k} - \alpha_j(1-\eta)(D^R_{j\to k})^2 \\
&= \alpha_j(1-\eta)D^{R,-1}_{k\to j}D^R_{j\to k} - \alpha_j(1-\eta)(D^R_{j\to k})^2 \\
&= \alpha_j(1-\eta)D^R_{j\to k}(D^{R,-1}_{k\to j} - D^R_{j\to k})
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{\partial \varepsilon^{-1}_{jj}}{\partial \tilde{p}_j} &= \varepsilon^{-2}_{jj}\alpha_j(1-\alpha_j)(1-\eta)(1-\eta), \\
\frac{\partial \varepsilon^{-1}_{jj}}{\partial \tilde{p}_k} &= (-1)\varepsilon^{-2}_{jj}\alpha_j\alpha_k(1-\eta)(1-\eta), \\
\frac{\partial m_j}{\partial \tilde{p}_j} &= 1-m_j, \\
\frac{\partial D^R_{j\to k}}{\partial \tilde{p}_j} &= 0, \\
\frac{\partial D^R_{j\to k}}{\partial \tilde{p}_k} &= \alpha_j(1-\eta)D^R_{j\to k}(D^{R,-1}_{k\to j} - D^R_{j\to k}).
\end{aligned}$$

Plugging the above to $\partial h_j/\partial \tilde{p}_j$ and $\partial h_j/\partial \tilde{p}_k$ gives

$$\begin{aligned}
\frac{\partial h_j}{\partial \tilde{p}_j} &= (-1)\frac{1}{\varepsilon_{jj}^2}\alpha_j(1-\alpha_j)(1-\eta)^2(1-m_k D_{j\to k}^R) - (1-m_j), \\
\frac{\partial h_j}{\partial \tilde{p}_k} &= \frac{1}{\varepsilon_{jj}^2}\alpha_k\alpha_j(1-\eta)^2(1-m_k D_{j\to k}^R) + (1+\varepsilon_{jj}^{-1})(1-m_k)D_{j\to k}^R \\
&\quad + (1+\varepsilon_{jj}^{-1})m_k\alpha_j(1-\eta)D_{j\to k}^R(D_{k\to j}^{R,-1} - D_{j\to k}^R).
\end{aligned}$$

The derivatives $\frac{\partial h_k}{\partial \tilde{p}_j}$ and $\frac{\partial h_k}{\partial p_k}$ are obtained symmetrically.

Application to the Staples/Office Depot Example

In the Staples/Office Depot example, we had $\alpha_1 = 0.473$, $\alpha_2 = 0.316$, $m_1 = 0.258$, $m_2 = 0.234$, $\varepsilon_{11} = -3.875$, $\varepsilon_{22} = -4.273$, $D_{1\to 2}^R = 0.599$, $D_{2\to 1}^R = 0.691$, and $\eta = 6.121$. The GUPPIs were $GUPPI_1 = 0.104$ and $GUPPI_2 = 0.137$.

The merger pass-through matrix calculated with the above formula is

$$M = \begin{bmatrix} 1.005 & 0.345 \\ 0.347 & 1.098 \end{bmatrix}.$$

Thus, the first-order approximation gives

$$\ddot{p} \approx M \cdot GUPPI = \begin{bmatrix} 0.152 \\ 0.187 \end{bmatrix}.$$